Utilizing Kriging Surrogate Models for Multi-objective Robust Optimization of Electromagnetic Devices

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***Abstract*—This paper presents a multi-objective optimization strategy assisted by three Kriging surrogate models: ordinary Kriging, first-order universal Kriging and second-order universal Kriging. These Kriging surrogate models present analytical optimization functions, which are accurate and fast to evaluate. A multi-objective particle swarm optimization (MOPSO) approach was chosen to deal with the robust optimization problem and to find out the set of robust optimal solutions that matches with the problem requirements. The experiments were performed in a robust version of the TEAM problem 22.**

***Index Terms* — Kriging surrogate models, particle swam optimization, robust optimization, TEAM 22.**

I. Introduction

The process of modeling the real system to be optimized involves several sources of uncertainties [1], [2]. For instance, precision can be lost when constructing a computational model of the optimization parameters; the environment (factors such as humidity, pressure, and temperature) may produce effects on the real system, that are difficult to be quantified; there may be measurement imprecision in the estimative of the objective functions and of the optimization parameters. A possible approach for dealing with uncertainty factors consists in constructing an optimization model, which includes additional parameters that reflect the uncertainty effect [3]. Therefore, because of adding the uncertainties, it is easy to produce a robust optimization in the objective and constraints functions. Robustness aims to find the robust solutions by the uncertainties for meeting the problem statement. [4].

The optimization process often requires approaches to alleviate the time spent for the simulations. Therefore, in this paper, a multi-objective optimization strategy using Kriging surrogate models, which is determined by comparing the accuracy of three different Kriging models. And the robust multi-objective PSO (MOPSO) is used to get the robust Pareto-optimal values of objective function. In addition, its behavior is investigated through application to the robust version of TEAM problem 22.

II. Robust Optimization

A. Multi-objective Robust Optimization

The robust formulation based on the worst case philosophy is presented as follows. Considering the design variables $\mathbf{x}$ and the uncertainty parameter $\mathbf{p}$, the vector of objective functions is defined as $\mathbf{f}(\mathbf{x}, \mathbf{p})$. Thus, the unconstrained robust minimization problem is defined as:

$$\min_{\mathbf{x}\in\mathbf{X}} \max_{\mathbf{p}\in\mathbf{P}} \mathbf{f}(\mathbf{x},\mathbf{p}) \tag{1}$$

The worst case vector of objective function $\mathbf{f}_{\mathrm{wc}}$ is defined as:

$$\mathbf{f}_{\mathrm{wc}i}(\mathbf{x},\mathbf{P}) = \max_{\mathbf{p}\in\mathbf{P}} \mathbf{f}_i(\mathbf{x},\mathbf{p}), \quad i=\{1,\cdots,m\} \tag{2}$$

By using the notation in (2), solving (1) consists in finding the set of robust solutions $\mathbf{X}^*$:

$$\mathbf{X}^* = \{\mathbf{x}^*\in\mathbf{X} \mid \nexists \mathbf{x}\in\mathbf{X}, \mathbf{f}_{\mathrm{wc}}(\mathbf{x},\mathbf{P}) \preceq \mathbf{f}_{\mathrm{wc}}(\mathbf{x}^*,\mathbf{P})\} \tag{3}$$

where the symbol $\preceq$ means $\mathbf{f}_{\mathrm{wc}}(\mathbf{x}, \mathbf{P}) \le \mathbf{f}_{\mathrm{wc}}(\mathbf{x}^*,\mathbf{P})$ and $\mathbf{f}_{\mathrm{wc}}(\mathbf{x}, \mathbf{P}) \ne \mathbf{f}_{\mathrm{wc}}(\mathbf{x}^*,\mathbf{P})$.

The robust Pareto front $\mathbf{Y}^*$ is the image of $\mathbf{X}^*$, considering the worst case performance of the elements from $\mathbf{X}^*$, then:

$$\mathbf{Y}^* = \mathbf{f}_{\mathrm{wc}}(\mathbf{x}^*,\mathbf{P}), \quad \mathbf{x}^*\in\mathbf{X}^* \tag{4}$$

The robust Pareto front $\mathbf{Y}^*$ is composed by worst case points as shown in (4). Therefore, $\mathbf{Y}^*$, or some of its elements, may be outside of the robust objective space f($\mathbf{X}$, $\mathbf{P}$),

$$f(\mathbf{X},\mathbf{P}) = f(\mathbf{x},\mathbf{p}), \quad \mathbf{x}\in\mathbf{X}, \mathbf{p}\in\mathbf{P} \tag{5}$$

However, this does not affect our proposed method since the worst case points are used only as reference points to guide the search to the non-dominated robust solution set.

Some studies utilized an evolutionary approach to solve a constrained multi-objective electromagnetic design problem. However, the computational cost is very high. In this paper, the Kriging surrogate models will be applied to improve the efficiency of the optimization process[5].

B. Kriging Surrogate Models

In the Kriging method, the response function of a deterministic computer experiment is given by

$$Z(\mathbf{x}) = \boldsymbol{f}^T(\mathbf{x})\boldsymbol{\beta} + \varepsilon(\mathbf{x}) \tag{6}$$

where $\mathbf{X}$ is the position vector with n dimension, $\boldsymbol{f}(\mathbf{x})=[f_1(\mathbf{x}), f_2(\mathbf{x}),\cdots, f_k(\mathbf{x})]^T$ is the regression function, $\boldsymbol{\beta}=[\beta_1, \beta_2, \cdots, \beta_k]^T$ is the unknown vector of regression coefficients, $\boldsymbol{f}(\mathbf{x})^{\mathbf{T}}\boldsymbol{\beta}$ is called a drift function showing the average behavior of response $Z(\mathbf{x})$, and $\varepsilon(\mathbf{x})$ is a random error term with $E[\varepsilon(\mathbf{x})]=0$.

According to the different drift functions, Kriging models are generally divided into Simple Kriging, Ordinary Kriging, and Universal Kriging. Universal Kriging is a non-stationary geo-statistical method and its drift function is a general linear function. Due to complexity of equation calculation, it is seldom investigated. The universal Kriging equations are generally expressed as follows:

$$\begin{cases} \sum_{i=1}^{N} \lambda_i f_k(\mathbf{x}_i) = f_k(\mathbf{x}), k = 0, \cdots, m+1 \\ \sum_{i=1}^{N} \lambda_i Cov[Z(\mathbf{x}_i), Z(\mathbf{x}_j)] + \sum_{k=0}^{m+1} \delta_k f_k(\mathbf{x}_j) = Cov[Z(\mathbf{x}), Z(\mathbf{x}_j)], j = 1, \cdots, N \end{cases} \quad (7)$$

where λ_i is unknown coefficient, δ_k is Lagrange multiplier, and $f_k(\mathbf{x}_j)$ is the basis function, which is normally a polynomial set for universal Kriging.

According to the different k values of the basis function, the Kriging method is classified into the ordinary Kriging (zero-order universal Kriging), first-order universal Kriging, second-order universal Kriging.

The stochastic component has a mean value of zero and following covariance:

$$Cov[Z(\mathbf{x}_i), Z(\mathbf{x}_j)] = \sigma^2 \mathbf{R}\left[R(\mathbf{x}_i, \mathbf{x}_j)\right] \quad (8)$$

In general, for Kriging, the covariance function should be also defined. In this paper, for samples $\mathbf{x}_i$ and $\mathbf{x}_j$, the Gaussian covariance function is defined as follows:

$$R_z(\mathbf{x}_i, \mathbf{x}_j) = \exp[-\sum_{d=1}^{D} \theta_d (x_{id} - x_{jd})^2] \quad (9)$$

where $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_D]^T$ is a covariance parameter vector, which influences the effect of covariance function along direction.

For the Gaussian covariance function, the best covariance parameter $\boldsymbol{\theta}$ can be obtained by Maximum Likelihood Estimation. So the unknown coefficients in Kriging surrogate models are determined based on all sampling points and objective values. Then the estimator at unknown point $\mathbf{x}$ is obtained by a linear combination of the sampling values as shown in (10). Then, the MOPSO is used to search for Pareto-optimal solutions of the response model constructed by Kriging surrogate models.

$$\hat{Z}(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i Z(\mathbf{x}_i) \quad (10)$$

Once the Kriging surrogate models with Latin Hypercube Sampling (LHS) are constructed, during optimization, the performance analysis can be applied directly to the approximate response surface so that the computational cost is reduced. Therefore, the proposed optimization algorithm is naturally suited for application in electromagnetic design optimization.

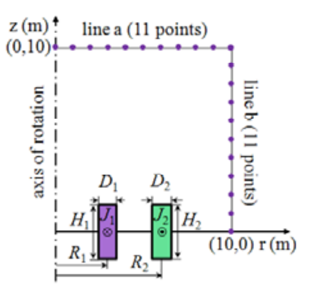

Fig.1. Configuration of the SMES device.

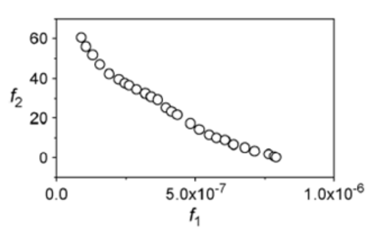

Fig.2. Pareto front by the deterministic multi-objective problem.

TABLE I
PARAMETERS FOR TEAM 22

Unit[m]	R_1	R_2	H_1	H_2	D_1	D_2	ΔH_2	ΔD_2
min	-	2.6	-	0.408	-	0.1	-	-
max	-	3.4	-	2.2	-	0.4	-	-
fixed	2.0	-	1.6	-	0.27	-	0.01	0.01

III. NUMERICAL EXAMPLE

The TEAM Problem 22 is an optimal design of a superconducting magnetic energy storage device to achieve the stored energy of E_0=180 MJ with minimal stray field. In this paper, the version of three design variables is investigated. The robust optimal design problem is formulated as [5]:

$$\begin{aligned} &\min_{\mathbf{x}\in\mathbf{X}} \mathbf{f}_1(\mathbf{x}, \mathbf{P}') : B_{stray}^2 \\ &\min_{\mathbf{x}\in\mathbf{X}} \mathbf{f}_2(\mathbf{x}, \mathbf{P}') : \left\| \mathrm{E} - E_0 \right\| \\ &\text{s.t.} \quad \mathbf{g}(\mathbf{x}, \mathbf{P}') : \left\| \mathbf{J} \right\| + 6.4 \left\| \mathbf{B}_{\max} \right\| - 54 \le 0 \end{aligned} \quad (11)$$

where $\mathbf{g}_{wc}(\mathbf{x}, \mathbf{P}^{'})$ is measured in A/mm^2. The first objective quantifies the stray field. The second objective measures the perceptual deviation from E_0 to computed energy E. Finally, the constraint function is addressed to ensure the quench condition. In this paper, uncertain variables are H_2 and D_2, a fixed uncertainty is 0.01m for $\mathbf{P}^{'}$ in Table I, ΔH_2 and ΔD_2 is the uncertainty. The main goal of this paper is to find the set of minimum $\mathbf{X}^*$ in (11) and the robust Pareto front associated with $\mathbf{X}^*$.

Fig.1 shows the parameters for TEAM 22, and Fig. 2 shows the Pareto front by the deterministic multi-objective problem. The discussion of detailed robust optimization results with the presence of uncertainties in variables H_2 and D_2 will be presented in the full paper.

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