Multiobjective Topology Optimization with Ant Colony Systems in Applied Electromagnetics

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***Abstract*—In this paper, a multiobjective Ant Colony System that would be suitable for multiobjective topology optimization problems in electromagnetic design is developed and tested in two case studies of design: a C-core magnetic actuator and an Interior Permanent Magnet (IPM) machine. The results are promising and illustrate the potential of the proposed methodology.**

***Index Terms*—Ant colony optimization, Design optimization, Computational electromagnetics, Electromagnetic modeling, Actuators, Rotors.**

I. Introduction

In Topology Optimization (TO) methods the topology and shape is not fixed *a priori*, on the contrary, it is possible to obtain designs by distributing the material in the design region, and the optimization process searches for the best shapes in some sense.

Since pioneering works by Bendsoe and Kikuchi [1], TO methods have been applied to various physical systems [2]. The first TO for designing electromagnetic devices was presented by Dyck and Lowther in 1996 [3]. Choi and Yoo proposed a method to TO for the simultaneous design of permanent magnets (PM) and ferromagnetic materials (FM) and applied it to the optimization of a C-core actuator and a magnetostrictive sensor [2]. Batista et al. presented a graph representation of the design space that is suitable for the ant colony optimization (ACO) method in TO problems and investigated the problem associated with the maximization of the torque of a C-core magnetic actuator with multiple materials [4].

The ACO was introduced by Dorigo in 1992 [5] and has been used to solve many combinatorial optimization problems, including multiobjectives ones [6]. It is based on the foraging behavior of some real ants that deposit pheromone along their paths when returning to his nest after successfully finding a food source. This pheromone model is used to construct candidate solutions and to bias future sampling toward high quality solutions.

In this paper, we develop a multiobjective ACO that would be suitable for multiobjective topology problems in electromagnetic design. With a multiobjective approach, the designer can identify and analyze trade-off topologies for the problem.

II. Multiobjective Ant Colony Optimization

A. Problem Statement

The design space of a TO problem is represented by a finite and bounded d-dimensional subset $\Omega \subset \mathbb{R}^d$, with $d = 2$ or 3, in which $\boldsymbol{c} \in \Omega$ denotes a cell within this geometric space. Each cell $\boldsymbol{c}$ is associated to a possible state within the set $S = \{1, \ldots, n\}$, being $\xi(\Omega) : \mathbb{R}^d \mapsto S^{|\Omega|}$ a function that attributes a state $e \in S$ to each cell. By considering the state of a given cell as the material properties at that point, then the general multiobjective TO problem can be defined as:

$$\xi(\Omega)^* = \arg\max_{\xi} \quad f_1(\xi(\Omega)), \ldots, f_M(\xi(\Omega)) \tag{1}$$

$$\text{Subject to :} \begin{cases} \xi(\Omega) \in S^{|\Omega|} \\ \text{Problem constraints} \end{cases} \tag{2}$$

where $f_i(\xi(\Omega)) : S^{|\Omega|} \mapsto \mathbb{R}, \quad \forall\ i = 1, \ldots, M$, are the objective functionals that are to be maximized, and the problem constraints are mathematical representations of the system requirements and limitations [4].

B. Proposed approach

The proposed ant colony algorithm uses the graph representation of the discretized design space, with each route corresponding to a given distribution of material, as in [4]. In order to updade the pheromone model for the multiobjective TO, the method also employs the Pareto dominance ranking (considering the ants in an external archive) and information about the time the ants were inserted into the archive. To improve smoothness the filtering strategy sugested in [4] is maintained. Two other mechanisms are proposed in this work: one to increase the smoothness and another to increase the diversity of the topologies and to spread the Pareto front. The first one takes each topology that generates a point in the Pareto front and creates a new topology (ant) by, randomly, applying (k times) a mask 2×2 and changing the materials to the value that appears more often in each 2×2 mask.The second mechanism consists in performing the possibles permutations of the three materials in one of the best topologies for each objective. These three mechanisms (filtering strategy, random smoothness and permutation of materials) should be applied in different cycles to obtain better results.

The Algorithm 1 describes the suggested multiobjective ACO, in which a variation in the quantity of pheromone is $\Delta\tau_{ij}^{(k,e_u)} = \Theta(k)Q$ if the kth ant in the updated archive uses edge e_u in its tour between time t and $t+q$, and $\Delta\tau_{ij}^{(k,e_u)} = 0$ otherwise, where Q is a constant and $\Theta(k)$ is a function proportional to the Pareto rank of the kth ant and inversely proportional to the time that this ant was inserted into the archive. A complete description of the method is left for the full paper.

Algorithm 1: Multiobjective Ant Colony Optimization

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1  Input: $c_{ij} \in \Omega$, n
2  begin
3      Initialize archive;
4      Initialize pheromone intensities $\tau_{ij}^{e_u}(t=0) = 1/n$;
5      Calculate the initial transition probabilities;
6      while maximum number of cycles not achieved do
7          foreach ant k=1, ..., m do
8              Perform a tour over the design space using $p_{ij}^{k,e_u}(t)$
           end
9          Apply filtering strategy at every 25 cycles;
10         Create extras ants by permutation of materials when
           $mod$(number of cycle, 25) $\equiv$ 5;
11         Create extras ants by random suavization of topologies of points in
           pareto front when $mod$(number of cycle, 25) $\equiv$ 10;
12         Evaluate objectives for all new ants
13         Atribute a Pareto rank to all objective points;
14         Update archive;
15         Update pheromone trail;
16         Set $\tau_{ij}^{e_u}(t+q) = \tau_{min}$ for those values below $\tau_{min}$;
17         Update transition probabilities;
       end
   end
18 Output: Estimate of the Pareto front and topologies
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The initial transition probabilities, in line 5, are calculated by $p_{ij}^{k,e_u}(t) = [\tau_{ij}^{e_u}]^{\alpha} / \sum_{u=1}^{n} [\tau_{ij}^{e_u}]^{\alpha}$.At line 12, the update is done by $\tau_{ij}^{e_u}(t+q) = (1-\rho)\tau_{ij}^{e_u}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^{(k,e_u)}$ and the transition probabilities are updated, in line 14, according to $p_{ij}^{k,e_u}(t) = [\tau_{ij}^{e_u}]^{\alpha} / \sum_{u=1}^{n} [\tau_{ij}^{e_u}]^{\alpha}$.

III. Cases Studies

The problem investigated consists in the optimization of a c-core magnetic actuator, see [2] and [4], which is composed of three main parts: the armature and the yoke solid blocks of ferromagnetic material (pure iron); and the design domain, which is discretized into a 20×10 square grid. The objectives are to maximize the x-directional attractive force on the armature, expressed in terms of the torque; and to minimize the volume of material. The second case is a topological design problem of the rotor of an IPM machine [7], [8]. Results of this design problem will be presented in the extended version.

IV. Results

To solve the problem defined in section III, the algorithm was set up as follows: number of ants $m = 20$, pheromone evaporation rate $\rho = 0.85$, and $Q = 100$. Furthermore, we have considered $\alpha = 1$, $\tau_{min} = 0.05$ and a maximum number of cycles $nc_{max} = 1000$. The maximum found for the force was 950.83 Nm (with pecentage of volume of PM equals to 50.5%) while the minimum pecentage of volume of PM was 1% (with force 0.02 Nm). These results are shown in Fig. 1

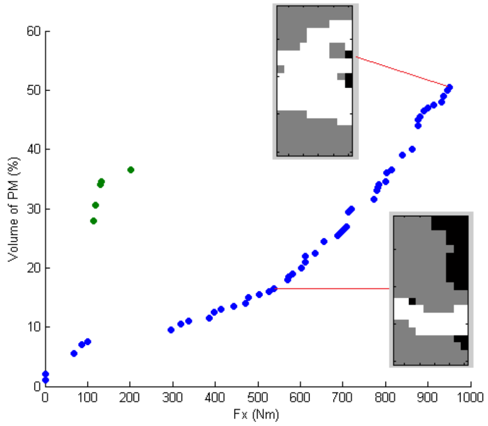

Figure 1. Pareto frontiers of cycles 1 and 1000 and two of the topologies found. White:PM; Gray:low-carbon steel; Black:air.

V. Conclusions

The results for the optimization of the C-core magnetic actuator are compatible with the studies in [2], [4], which show the adequacy and potential of the approach to multiobjective electromagnetic topology optimization. In the final version of this paper, the proposed ACO will be also used for the optimization of an IPM machine design.

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