Finite Element Based Circuit Model Approach for Skewed Electrical Machines

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***Abstract*—This work presents a finite element (FE) based model approach for permanent magnet machines (PMSM) and wound rotor induction machines (WRIM) taking skewing of the rotor or the stator into account. These models utilize look-up tables created by two dimensional (2D) FE simulations in advance. Inspired by the FE multi-slice technique multiple evaluations of these look-up tables are performed to simulate skewing. The resulting novel models are equivalent to 2D FE multi-slice models in all but interpolation errors but at much lower simulation cost.**

***Index Terms*— Computational modeling, finite element methods, induction machines, numerical models, permanent magnet motors.**

I. Introduction

In the design stage of electrical drive chains multi body simulations of the whole mechanical power train are used for structural analysis, bearing design, shaft dynamics or noise and vibration harshness (NVH) investigations. All of those simulation tasks need accurate models of electrical machines covering torque harmonics and saturation effects in the whole operating range of the machine. The well-known fundamental wave models are not able to satisfy the required model depth. Finite element models have this capability but the simulation cost of such models precludes their application in multi body simulations.

However, the FE based model approach combines the model depth of FE models with very low simulation cost. This is achieved by using look-up tables created by 2D magneto-static FE simulations in advance for describing the machine behavior. These look-up tables are parameterized by the rotor position and the machine currents and thus describe the 2D magneto-static machine behavior. Therefore, three dimensional (3D) effects like the influence of the end windings or skewing are not covered in principle. However, for a sufficiently high number of sampling points, the interpolation error can be minimized and this model approach becomes equivalent to the FE model. Several publications on this topic have already been published, for example [1] to [6].

Within the presented work an extension of this model approach for PMSMs and WRIMs is presented. Skewing is taken into account by multiple evaluations of the look-up tables at different rotor positions but with the same currents according to the 2D FE multi-slice technique [7], [8]. Each look-up table evaluation can be assumed to correspond to a single slice. The simulation time for this novel approach is significantly smaller than a FE multi-slice simulation. Furthermore, this model approach can be used for optimizing the skewing angle during the design stage of electrical machines.

II. FE-Based Model Approach

In this section a short introduction to the FE-based model approach for PMSMs as described in [5] is given to explain the functionality of this technique. The phase voltage v_A of phase A for a PMSM in Y-connection and with isolated star point can be written as

$$v_A = i_A R_{Cu} + \frac{d\Psi_A\left(\alpha_{Rot}, i_d, i_q\right)}{dt} \qquad (1)$$

with the phase current i_A, the ohmic resistance R_{Cu} and the phase flux linkage Ψ_A as function of the rotor position α_{Rot}, the direct current i_d and the quadrature current i_q.
The time derivative of the flux linkage Ψ in (1) can be evaluated by

$$\frac{d\Psi\left(\alpha_{Rot}, i_d, i_q\right)}{dt} = \frac{\partial\Psi}{\partial\alpha_{Rot}}\frac{d\alpha_{Rot}}{dt} + \frac{\partial\Psi}{\partial i_d}\frac{di_d}{dt} + \frac{\partial\Psi}{\partial i_q}\frac{di_q}{dt} \qquad (2)$$

using a look-up table for the phase flux linkage and an interpolation method that allows a direct evaluation of the partial derivatives. Finally, the complete machine model can be written with phase to phase quantities as

$$\begin{bmatrix} \frac{\partial\Psi_{AB}}{\partial i_d} & \frac{\partial\Psi_{AB}}{\partial i_q} \\ \frac{\partial\Psi_{BC}}{\partial i_d} & \frac{\partial\Psi_{BC}}{\partial i_q} \end{bmatrix} \begin{pmatrix} \frac{di_d}{dt} \\ \frac{di_q}{dt} \end{pmatrix} = \begin{pmatrix} v_{AB} \\ v_{BC} \end{pmatrix} - \begin{pmatrix} i_{AB} \\ i_{BC} \end{pmatrix} R_{Cu} - \begin{pmatrix} \frac{\partial\Psi_{AB}}{\partial\alpha_{Rot}} \\ \frac{\partial\Psi_{BC}}{\partial\alpha_{Rot}} \end{pmatrix} \frac{d\alpha_{Rot}}{dt}. \qquad (3)$$

Due to symmetry Ψ_{BC} in (3) is equal to Ψ_{AB} except for a shift of 120 electrical degrees. This system of nonlinear ordinary differential equations is solved iteratively using the Crank-Nicolson method.

The machine torque T is given directly by an additional look-up table

$$T = f\left(\alpha_{Rot}, i_d, i_q\right). \qquad (4)$$

This PMSM model approach needs two look-up tables Ψ_{AB} and T, both parameterized with α_{Rot}, i_d and i_q. These look-up tables are created by magneto-static FE-simulations. Therefore, any eddy current effects like eddy currents within the permanent magnets or skin effect are not considered. However, this model approach is fast at acceptable memory demand and therefore well suited for modeling electrical machines in multi-body simulations.

The model approach for WRIMs works in a similar manner. However, the higher number of coil systems leads to a higher number of state variables and more look-up tables. A detailed description of the FE based WRIM model can be found in [6].

III. Consideration of Skewing

A 2D multi-slice FE model is sufficient for taking the skewing of the rotor or the stator into account. This simulation technique uses several 2D FE models of the machine (slices). These slices differ only in the relative rotor position relating to the stator reference system. All slices are coupled in a way that the currents are enforced to be the same in the corresponding coils of each slice, as shown in Fig. 1.

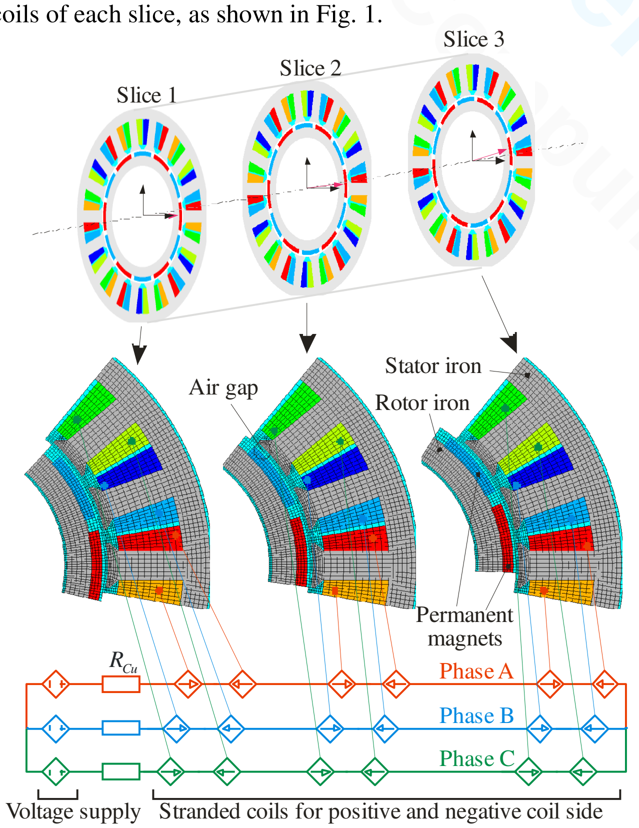

Fig. 1. 2D FE multi-slice model of a PMSM with three slices. The coupling of the slices is done with circuit elements to ensure the same coil current in the corresponding coil of each slice. Due to symmetry only a segment of the whole machine is modeled.

The same method can be easily applied to the FE-based model approach. Therefore, all look-up tables just need to be evaluated at different rotor positions with the same currents. Finally, these results need to be summed up, taking the length per slice l_n and the angular offset per slice α_n into account. This leads for Ψ to

$$\Psi = \frac{1}{l_{Ref}} \sum_{n=1}^{N_{Slice}} l_n \Psi\left(\alpha_{Rot} + \alpha_n, i_d, i_q\right) \qquad (5)$$

with the reference length of the look-up table l_{Ref} and the number of slices N_{Slice}. Due to the linearity of the derivative operator, the same summation is also valid for the interpolated partial derivatives used in (3).

Although (5) refers to PMSMs, this method works in the same manner for WRIMs.

IV. Advantages of the Method

One advantage of this extended model approach is that no additional look-up tables are needed compared to the not skewed model. All necessary changes only affect their evaluation.

Another advantage is the fast evaluation of the look-up table interpolation as shown in [5] and [6]. Therefore, this approach is not limited to as few slices as typically used for 2D multi-slice simulations. In combination with the slice parameters l_n and α_n, a very detailed modeling of the electrical machine in axial direction is realizable. For example the whole lamination stack including cooling slots could be modeled with one slice per laminated sheet package. Hence, this model approach can be also used in the design stage of electrical machines for optimizing the skewing angle. This could replace time consuming FE multi-slice simulations.

V. Validation

For the validation of the PMSM-model as well as the WRIM-model introduced in this work, several comparisons with FE multi-slice simulations have been carried out. The result of this comparison will be shown in the full version of this work.

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