# Equivalent Circuit Parameters Calculation of Induction Motor by Finite Element Analysis

Zaixun Ling, Libing Zhou, Siyuan Guo, Yi Zhang

State Key Laboratory of Advanced Electromagnetic Engineering and Technology,

Huazhong University of Science and Technology

Wuhan Luoyu Road 1037, 430074, China

lingzaixun@163.com

Abstract—This paper presents a accurate calculation method to extract parameters of induction motor (IM) from finite element field solutions. A 2D steady state AC magnetic application of IM is used in the accurate simulation. and in order to separate the leakage reactance, the frozen permeability method is used in the magneto static field simulation for the slot leakage reactance calculation. The end effects are considered by field-coupled circuit elements and corrections can be introduced to take rotor skew into account. Then, the determination of reactance as a function of saturation level is discussed in particular. An application example is provided to confirm the accuracy of the proposed approach.

*Index Terms*—finite element method; parameters calculation; leakage reactance separation; saturation magnetization.

## I. INTRODUCTION

The classical equivalent circuit method is a traditional tool for predicting the behavior of IM and optimizing design. This method, however, does not give enough accuracy because it assumes linearity of the iron core and derived from the analytical solutions under basic simplified assumptions; Thus, many correction factors are needed to adjust the design solution to the testing results, and significant errors may be caused when analyzing different specifications of motor [1]. The result obtained in start-up process also differ from rated parameters.

With the development of numerical modeling techniques, the finite element method (FEM) is well suited for the study of electromagnetic device. A method based on the use of complex 2D finite element solutions for accurately predicting the steady state performance is presented in [2]-[3], the saturation effect is considered by introducing an effective reluctivity, but the end effects is neglected in finite element solving process. Then, Field-coupled circuit elements is used to take end effects into consideration in [3]-[4]. Among the parameters of IM, resistances of stator windings and rotor bars is easy to be obtained, but the excitation reactance and leakage reactance are difficult to be calculated by FEM. Literature [3]-[5] calculate excitation reactance by no-load back electromotive force (EMF). However, due to the saturation of main field and the influence of rotor response magnetic field, the excitation reactance is variable at different operating point. The determination of leakage reactance is usually be carried out under short circuit conditions with a assumption that the stator leakage reactance and rotor leakage reactance are equal [4]-[5]. Therefore, the results of leakage reactance are lack of accuracy. The author makes a meaningful discussion about the variation of leakage reactance in no-load condition as a function of magnetizing current in [6]. But the slot leakage

reactance and the harmonic leakage reactance are still not extracted yet.

In this paper, a more accurate method for computing the parameters of IM is presented. By using the vector addition method and the Fourier analysis method, the excitation reactance and leakage reactance can be obtained with more accuracy. Furthermore, the slot leakage reactance and the harmonic leakage reactance are separated precisely from leakage reactance in this paper. Then a prototype machine is built to confirm analysis results.

#### II. MODELING PROCEDURE

### A. Fundamental Equations

Fig. 1 shows the equivalent circuit of IM. The stator phase resistance  $R_I$ , skew reactance  $x'_{sk}$ , the ending effects  $x_{I\varepsilon}$  and  $x'_{2\varepsilon}$  are calculated by analytical solution.

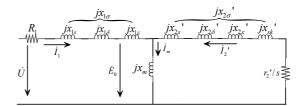


Fig. 1. Equivalent circuit of the IM

The rotor bar resistance  $r_2$ ' can be calculated by joule loss in bars and end windings

$$r_{2}' = k_{e}k_{i}(P_{rb} + P_{rr})/I_{bar}^{2}.$$
 (3)

As Fig. 2 shows, the Fourier analysis method is applied to decompose the air-gap flux density into the basic wave and a series of harmonic wave.

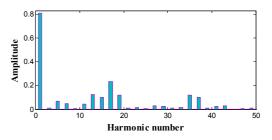


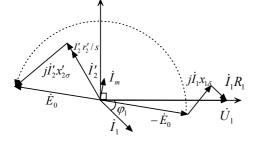
Fig. 2. Spectrum analysis of flux density through the air-gap The EMF  $E_0$  per phase is induced by fundamental field

$$E_0 = -\sqrt{2\pi} N_1 k_{w1} f_1 (2B_{m1} L_{ef} \tau / \pi) .$$
 (4)

Then, rotor leakage reactance  $x'_{2\sigma}$  can be calculated by

$$x_{2\sigma}' = \sqrt{\left(E_0 / I_2'\right)^2 - \left(r_2' / s\right)^2} \ . \tag{5}$$

According to their vector relation, the phase of EMF and the stator leakage reactance  $x_{1\sigma}$  are obtained from Fig. 3.



## Fig. 3. Vector diagram of IM equivalent circuit

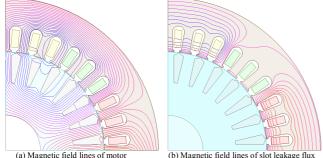
When  $x_{1\sigma}$  and  $x_{2\sigma}$ ' are solved, the magnetizing reactance  $x_m$  can be obtained by calculating the reactive power per phase

$$x_m = E_0^2 / Q_m = E_0^2 / (Q_1 / 3 - x_{\sigma 1} I_1^2 - x_{\sigma 2} ' I_2'^2) .$$
 (6)

## B. Separating the leakage reactance

In general, the leakage reactance can be decomposed into five principle components: Slot leakage reactance, end winding leakage reactance, harmonic leakage reactance, zigzag leakage reactance and skew leakage reactance [7]. The zig-zag leakage flux in IM can be neglected due to the small air gap. In this paper,  $x'_{sk}$ ,  $x_{1\varepsilon}$  and  $x'_{2\varepsilon}$  are all calculated by analytical solution. So the leakage flux in finite element field solution is only consist of slot leakage flux and harmonic leakage flux, but it is still hard to separate them.

In order to calculate the slot leakage reactance, the whole procedure may be described in terms of the following steps:



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Fig. 4. The magnetic field distribution in IM **Step 1**. A 2D steady state AC magnetic field is first carried out, record the real component of stator wingding current  $I_a$ ,  $I_b$ ,  $I_c$ , and total current in each bar.

**Step 2**. Set the solution type of design as magneto static. The excitation of field is imposed according to the result of the previous step. Freeze the permeability of each element in model when the design is solved.

**Step 3**. Create a new design by copying and pasting the model in Step 2, so that the mesh between two linked designs is the same. Use the permeability that has been frozen from Step 2 as stator core's permeability. Set the relative permea-

bility of rotor, shaft and bars as 1e-7, that means those regions are non-magnetic. Then, the winding A is excited by 1A, wingding B and winding C are excited by 0.5A, all rotor bars are open. Fig. 4 shows the stator slot leakage flux in IM. Calculate the magnetic energy in whole domain, then, the stator slot leakage reactance of phase A is obtained by

$$x_{s1} = 2\pi f \left( 2W_m / I^2 \right). \tag{6}$$

This step is repeated by the excitation of B and C windings to calculate the average value of three leakages.

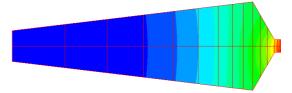


Fig. 5. The segmentation of rotor bar and the distribution of current density

**Step 4.** The process of rotor slot leakage reactance determination is similar with above procedure. But due to the skin effect, the current density on conducting bar interface is not homogeneous. So, each bar could be divided into a number of sections, Fig. 5. Record the current in each section and stator winding current as the excitation of magneto static field. Then do Step 2 and Step 3 again. One should pay attention that the non-magnetic region in Step 3 is replaced by stator, coil and wedge sector. The current excitations are imposed to the sections of one bar according to the result of this Step. Calculate the slot leakage reactance of each rotor bar, then the average slot leakage reactance  $x'_{2s}$  is obtained by

$$x_{s2}' = k_e k_i 2\pi f \sum_{j=1 \sim z_2/p} p(2W_{mbj} / I_{bj}^2) / Z_2.$$
 (6)

When slot leakage reactance is obtained, the harmonic leakage reactance can be calculated by  $(x_{1\sigma} - x_{1s})$  and  $(x'_{2\sigma} - x'_{2s})$ .

#### III. RESULTS AND CONCLUSION

Because of the space restriction, the detailed derivation process will be presented in full paper.

#### REFERENCES

- A. Boglietti, A. Cavagnino, M. Lazzari, "Computational Algorithms for induction motor equivalent circuit parameter determination—Part I," *IEEE Trans. Ind. Electron.*, vol. 58, no 9, September 2011.
- [2] A. Yahiaoui, F. Bouillault, "Saturation effect on the electromagnetic behaviour of an induction machine," *IEEE Trans. on Magn.*, vol.31, no.3, September 1995.
- [3] P. Zhou, J. Gilmore, Z. Badics, Z. J.Cendes, "Finite Element analysis of induction motors based on computing detailed equivalent circuit parameters," *IEEE Trans. on Magn.*, vol.34, no.5, September 1998.
- [4] X. H. Wang et al., "Performance analysis of single-phase induction motor based on voltage source complex finite-element analysis," *IEEE Trans. on Magn.*, vol. 42, no.4, pp. 587-590, 2006.
- [5] A. Stermecki, et al., "Calculation of load-dependent equivalent circuit parameters of squirrel cage induction motors using time-harmonic FEM," *Proc. ICEM*. Vilamoura. Portugal. September 2008.
- [6] D. Dolinar, R. De Weerdt, R. Belmans, E.M. Freeman, "Calculation of two-axis induction motor model parameters using finite elements," *IEEE Trans. on Energy Conversion.*, vol.12, no.2, June 1997.
- [7] T. A. Lipo, Introduction to AC Machine Design. WI: Wisconsin Power Electronics Research Center, Univ. of Wisconsin, 1998, vol. 4.