Extended Finite Element Method for Electric Field Analysis

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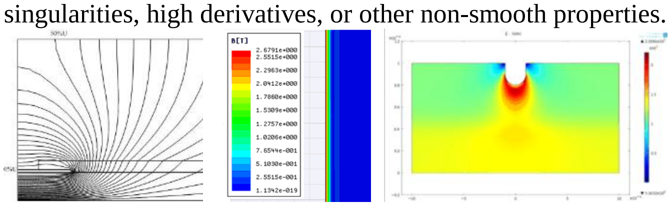
***Abstract*—This paper presents the fundamental principle of the extended finite element method (XFEM) for the analysis of electric field in both dielectric and conductors. This method provides an accurate approximation for locally non-smooth features within finite elements, such as singularities, discontinuities, and high derivatives. An alternative enrichment function is introduced to improve the approximation shape function of the classical finite element method (CFEM). In XFEM, the non-smooth solutions are modeled independent of the mesh and the level set method is employed to describe the interfaces among different materials in one finite element. To demonstrate the advantages, the XFEM is compared with CFEM, analytical method and experiments by solving some electric field problems.**

***Index Terms*—Electric field, extended finite element method, level set, thin layer, crack tip.**

I. Introduction

In a large number of electromagnetic applications, the quantities of electromagnetic field change rapidly over length scales which are small in comparison to the solution domain. Such examples include the high and steep electric field distribution at the cable termination, the high frequency magnetic field distribution due to the skin-effect in a solid conductor [1], and the electric field containing movable charges very closely to the sharp tip of conductor, as shown in Fig. Fig. 1 (a), (b) and (c) respectively, etc. To model such phenomena, the solutions typically involve discontinuities, singularities, high derivatives, or other non-smooth properties.



(a) (b) (c)

Fig. 1 Singular electric field in cable termination (a), high frequency magnetic field due to skin effect (b) and singular electric field closed to sharp tip

In past decades, the extended finite element method (XFEM), which was firstly proposed by Belytschko, *et al* [2], provides a mesh-independent approximation for non-smooth problems. Aiming to the approximation of non-smooth solutions, the traditional approach is to employ the polynomial approximation, which depends on meshes that conform to discontinuities and are refined near singularities and high gradients [3]. However, in extended finite element method, the strategy is to enrich a polynomial approximation space such that the non-smooth solutions can be modeled independent of the mesh. The enrichment is realized by appending special shape functions, which is not necessarily polynomial and matches the assumed characteristics of the solution and thus ensure good local approximation, to traditional polynomial approximation,. In XFEM, a locally enrichment function, described as a discontinuous shape functions, is adding to the classical FEM through a partition of unity method.

II. Principle of Extended Finite Element Method

The classical FEM depends on the construction of meshes aligning with the interfaces and boundaries. The meshes are refined near domains which possess high variant field over very small space, discontinuity across an interface, etc. The accuracy is improved for smooth solutions, such as super convergent patch recovery [4].

In the XFEM, the interface between materials is not aligned with the edges of finite elements. The level sets can provide smoother optimal boundaries and material interfaces for topology optimization [5]. In this paper, level sets are introduced for the mathematical description of interface in the enrichment approximation. Fig. 2 pictured the interface described as zero level set function $\psi(\boldsymbol{X})=0$.

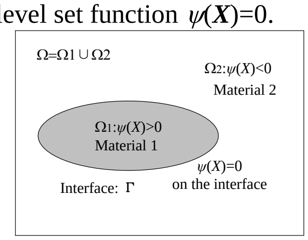


Fig. 2. The interface representation with level set function

In the discretized domain, I is the set of all nodes, I^* is the set of the enriched nodes, $I^* \in I$. The approximation of a potential function $u(\boldsymbol{X})$ may be defined as [2]

$$u^h(\boldsymbol{X}) = \sum_{i\in I} N_i(\boldsymbol{X}) u_i + \sum_{i\in I^*} N_i^*(\boldsymbol{X}) \cdot \left[\psi(\boldsymbol{X})\right] a_i \tag{1}$$

where the first term on the right-hand side is the standard FE approximation, the second the enrichment, which extends the standard FE approximation, i the index number of FE nodes contained in set $\boldsymbol{I}$, or the enriched nodes contained in $\boldsymbol{I}^*$, which is a subset of $\boldsymbol{I}$. N_i and N_i^* are standard FE shape functions, in general, $N_i = N_i^*$. The coefficients u_i belong to the standard FE part and a_i are additional nodal unknowns. The function $\psi(\boldsymbol{X})$ is called an enrichment function according to special and detailed knowledge for the solution problem. The products $N_i(\boldsymbol{X}) \cdot \psi(\boldsymbol{X})$ are local enrichment functions because their supports coincide with the supports of typical FE shape functions, leading to sparsity in the discrete equations.

Based on the number of enriched nodes in an element, an element may be cataloged into (i) the enriched element if all of its nodes are enriched and the interface are passing through the element; (ii) the FE element if none of its nodes is enriched, and (iii) the blending element if some or all of its nodes are enriched. No interface intersects the element.

In the XFEM, different element integration methods are designed for the discontinuity in the enriched element, high gradient and singular enrichments. For discontinuity, decomposition of element into sub-elements aligning with the interface is a useful option for enriched element integration. In each sub-element, the numerical integration, such as Gauss quadrature, should be applied and the element integration is equal to the sum of the integrations of sub-elements.

Based on the processing of classical finite element, the matrix equation of XFEM may be achieved as

$$\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{ua} \\ \boldsymbol{K}_{au} & \boldsymbol{K}_{aa} \end{bmatrix} \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{A} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_u \\ \boldsymbol{P}_a \end{bmatrix} \tag{2}$$

where, $\boldsymbol{U}$ and $\boldsymbol{A}$ designate the column vectors of nodal electrical potentials and additional unknowns, respectively. Other variables will be introduced in full paper.

III. Simulation Verification of XFEM

A. 1D electric field

Fig. 3 shows parallel plate electrode system containing three dielectrics, FEM model and XFEM model. The width of three dielectrics are l_1=10mm, l_2=1mm and l_3= l_4=8mm. The middle of the three dielectrics may be regarded as a very thin layer material because the width of middle dielectric is about 3.8% of that of the others. The permittivity of three dielectrics are $\varepsilon_1=\varepsilon_0$, $\varepsilon_2=2\varepsilon_0$ and $\varepsilon_3=3\varepsilon_0$, respectively. The imposed voltage is 10V. The electrical potential approximation may be written as

$$\varphi^h(x) = \sum_{i\in I} N_i(x)\hat{\varphi} + \sum_{i\in I^*} H_{\Gamma d}(N_i(x))\tilde{\varphi} \tag{3}$$

where, $H_{\Gamma d}(x)$ is the Heaviside jump function.

The related error comparison between CFEM and XFEM is listed in Table I. The meshes used in both XFEM and CFEM will be improved in the full paper and the computation complexity will be evaluated by comparison the results of XFEM with those of CFEM and analytical solutions.

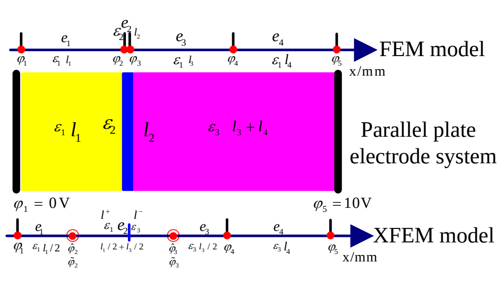


Fig. 3 Parallel plate electrode system containing three dielectrics

B. 2D electric field

The conducting current distribution in a non-uniform electric field will be analyzed by XFEM. The electrode structure, one 2D plate electrode system with crack, may be shown in Fig. 4, which may describe the field concentration effects of the crack tip. Fig. 4 also illustrates the experiment device for the measurement of electric field distribution according to the duality theory of static and steady electric fields. The electrical bridge is used to measure the electrical potential distribution. The accuracy and effectiveness of XFEM will be verified by comparison with experiment and CFEM.

TABLE I
The Comparison of Nodal Electrical Potential

X (mm)	XFEM φ (V)	CFEM φ (V)	Related Error (%)
5	3.15790	3.15790	0.00
10	6.31579	6.31579	0.00
11	6.63158	6.63158	0.00
15	7.47368	7.47368	0.00
19	8.31579	8.31579	0.00

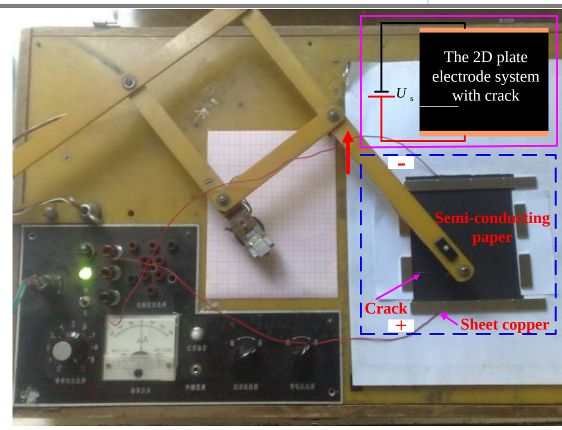


Fig. 4. The 2D plate electrode system with crack and the experiment device

IV. Conclusion

In this paper, the eXtended Finite Element Method for electric field analysis is presented. An enrichment function may improve the standard finite element approximation in the cases of discontinuities, singularities, high derivatives, or other non-smooth properties. One of the advantages of XFEM is that the interfaces do not align with the edges of FE meshes. The numerical simulations conclude that XFEM is able to simulate discontinuities and singularity behaviors of the electric field on a mesh that is independent from the interface location.

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