A Fast Frequency-domain Parameter Extraction Method Using Time-domain Finite-element Method

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***Abstract*—In the design of electric devices, usually it is required to extract the parameters of equivalent circuit in different frequencies. Traditional method is to solve the eddy-current field in frequency-domain and to calculate the parameters in post-processing. For obtaining the parameters in different frequencies, frequency-sweeping is required. It is time-consuming and inefficient. In this paper a novel method for the impedance extraction is presented. It only needs to solve the problem of a time-domain finite-element method (FEM) a few times, the impedances in all frequencies can be directly obtained. Numerical experiments show that it can reduce the computing time about 95%.**

***Index Terms*—Electromagnetic field, finite element method, frequency domain, impedance extraction, time domain.**

I. Introduction

Impedance extraction is one of the main purposes of eddy-current magnetic field computation [1-3]. In traditional methods the resistances are computed from eddy-current loss; and the inductances are computed from flux linkage or magnetic field energy [4]. The demerits of these methods are that special post-processing algorithms are required, and for parameters with different frequencies, the field matrix equation needs to be repeatedly solved many times.

In this paper a fast method to extract the equivalent circuit parameters of eddy-current magnetic field is presented. It is based on time-domain magnetic field circuit coupled FEM. With sinusoidal voltages applied to the terminals of windings, the currents in the windings are firstly computed and the impedance is then directly obtained. To overcome the problem that the system matrix equation needs to be solved for each frequency, a fast algorithm which is based on the time-domain solutions is presented. The merits of the proposed method are that:

(1) Only base solutions in time-domain need to be computed. If the parameters at different frequencies are required to be computed, a frequency sweeping process can be avoided. In a numerical experiment being studied, the computing time of the proposed method is less than 5% of that required if conventional frequency-domain method is used.

(2) No special post-processing algorithm is needed. All the effects in eddy-current field, including floating conductors which are not the circuit ports, internal circuits which are not the ports for parameter extraction, as well as the displacement current in the direction of the model depth can all be taken into account in the extracted lumped parameters.

II. Obtaining Field Solutions Using Time-domain FEM

Essentially the solutions with unit step functions are firstly obtained in time-domain, which are referred as base solutions. As a sinusoidal function can be considered as the addition and subtraction of many step functions, the solutions with sinusoidal function excitations at different operating frequencies can be easily and quickly obtained by the addition and subtraction of the base solutions.

After the response $i_{step}(t)$ of a step function $u(t-\tau)$ is obtained from time-domain, for the voltage source $v_s(t) = V_{sm}\sin(t)u(t)$, its solution is

$$i_{sin}(t) = v_s(t-\tau)u(t-\tau)i_{\text{step}}(\tau)\Big|_{\tau=0}^{\tau=\infty} + \int_0^\infty i_{\text{step}}(\tau)V_{sm}\omega\cos\omega(t-\tau)u(t-\tau)\mathrm{d}\tau, \quad (1)$$

III. Parameters of Frequency-domain

After the frequency-domain solutions of the field equations and circuit equations are available, the parameters can be extracted based on their voltages and currents in each windings. Supposing there are totally N stranded windings and solid conductors, the equivalent circuit matrix equation is:

$$\begin{Bmatrix} \dot{U}_{w(1)} \\ \dot{U}_{w(2)} \\ \vdots \\ \dot{U}_{w(N)} \end{Bmatrix} = \begin{Bmatrix} \dot{E}_{o(1)} \\ \dot{E}_{o(2)} \\ \vdots \\ \dot{E}_{o(N)} \end{Bmatrix} + \left\{ \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{21} & \cdots & R_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} + j\omega \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1N} \\ L_{21} & L_{21} & \cdots & L_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ L_{N1} & L_{N2} & \cdots & L_{NN} \end{bmatrix} \right\} \begin{Bmatrix} \dot{I}_{w(1)} \\ \dot{I}_{w(2)} \\ \vdots \\ \dot{I}_{w(N)} \end{Bmatrix}. \quad (2)$$

Its equivalent circuit is as shown in Fig. 1.

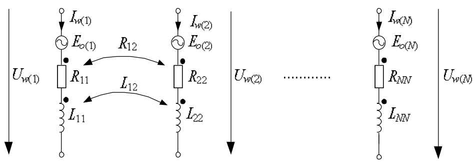

Fig. 1. The equivalent circuit of N windings

The procedure to compute the parameters is:

(a) Keep all sources of current densities and internal circuits, as well as boundary conditions; set all sources of stranded windings and solid conductors to be zero; compute the field in time-domain, then compute the back emf $\dot{E}_{o(i)}$ (i = 1, 2, …, N) in each stranded windings and solid conductors.

(b) Set all non-zero-value boundary conditions to zero; set all internal sources to zero; set a unit current source only in the i^{th} stranded winding or solid conductor; set the current sources in all other stranded windings or solid conductors to zero; compute the magnetic field – circuit coupled problem in time-domain and then compute the voltages $\dot{U}_{w(i)}$ (i = 1, 2, …, N) in each windings. The resistance and the inductance are:

$$R_{ij} = \mathrm{Re}\left(\frac{\dot{U}_{w(j)}}{\dot{I}_{w(i)}}\right). \qquad (j = 1, 2, \ldots, N) \quad (3)$$

$$L_{ij} = \frac{1}{\omega}\mathrm{Im}\left(\frac{\dot{U}_{w(j)}}{\dot{I}_{w(i)}}\right). \qquad (j = 1, 2, \ldots, N) \quad (4)$$

When each FE field is computed, the coefficient matrix of the system algebraic equation is kept the same; only the right hand side changes and only a multi right hand side (RHS) problem needs to be solved. By using the multi-RHS algebraic solvers, the computing time required to extract the parameters can be greatly reduced.

The following formulation will reveal the relationship among the parameters. If there is only excitation in the i^{th} stranded winding, the voltage of winding i can be obtained as follows:

$$\dot U_{w(i)} = \mathrm{Im}\left[-\frac{\frac{l}{S}\iint_{\Omega_i}\omega \dot A d\Omega}{\dot I_{w(i)}}\right]\dot I_{w(i)} + j\omega \mathrm{Re}\left[\frac{\frac{l}{S}\iint_{\Omega_i}\dot A d\Omega}{\dot I_{w(i)}}\right]\dot I_{w(i)} + \frac{1}{G_{w(i)}+j\omega C_{w(i)}}\dot I_{w(i)} \quad . \qquad (5)$$

The total voltage of the winding i is:

$$\dot U_{w(i)} = \dot E_{o(i)} + \sum_{j=1}^{N} R_{w(ij)}\dot I_{w(j)} + j\omega\sum_{j=1}^{N} L_{w(jj)}\dot I_{w(j)} + \frac{1}{G_{w(i)}+j\omega C_{w(i)}}\dot I_{w(i)} \quad . \qquad (6)$$

From (6) a precise equivalent circuit can be obtained.

IV. Numerical Experiment

A contactless transformer is analyzed to validate the proposed method. The equivalent circuit of the transformer can be regarded as a two-port network as shown in Fig. 2. The transformer is analyzed using time-domain FEM.

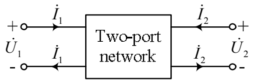

Fig. 2. Two-port network

In order to obtain the response under sinusoidal voltage excitation with different frequencies, the unit step function response is computed when the second port is open-circuit, which is shown in Fig. 3. The induced emf in the second port is also obtained. Then the current $\dot I_1$ and voltage $\dot U_2$ under sinusoidal voltage excitation with different frequencies are calculated according to (1) when second port is open-circuit.

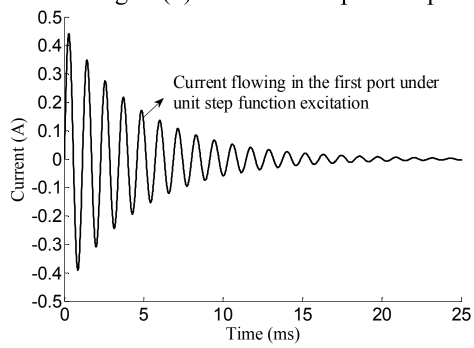

Fig. 3. Current in the first port under unit step function excitation when the second port is open-circuit

As an example, the following results are under the sinusoidal voltage excitation with 0.837 MHz as shown in Fig. 4 (blue line). Fig. 5 gives the current flowing in the first port (green line) and the induced emf in the second port (blue line) when the second port is open-circuit. According to the theory of the two-port network, the open-circuit input impendence (Z_{11}) can be calculated through the ratio between the voltage and current in Fig. 4, while the open-circuit transfer impendence (Z_{12}) can be calculated through the ratio between the emf and current in Fig. 5.

During the calculation, the phase difference (φ) between the voltage and current needs to be obtained through the time difference of the two curves over the zero point. It can be obtained by

$$\varphi = \left(2\pi\sum_{i=1}^{N}\left|t_U^i - t_I^i\right|\right) \Big/ N\cdot T \, , \qquad (7)$$

where N is the number of points over the zero; T is the cycle of the curve; t_U^i and t_I^i are the time of the voltage curve and current curve over the zero point, respectively. The more detailed results will be given in full paper.

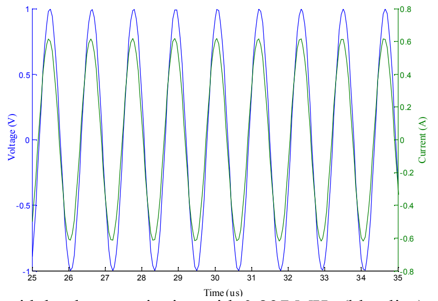

Fig. 4. Sinusoidal voltage excitation with 0.837 MHz (blue line) and current flowing in first port (green line) when the second port is open-circuit

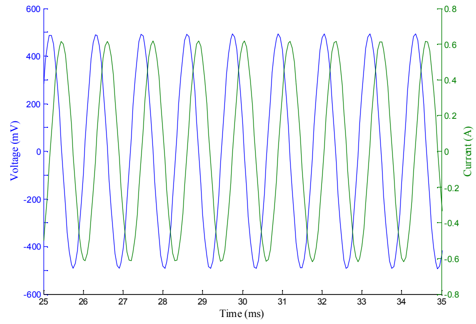

Fig. 5. Current flowing in first port (green line) and the induced emf in the second port (blue line).

V. Conclusion

The proposed frequency-domain parameter extraction method using time-domain FEM can quickly obtain the impedances over all operating frequencies. The time-domain FEM only needs to be analyzed a few times and then the parameters can be obtained based on the unit step function response. Then the parameters under different frequencies can be easily calculated according to the proposed formulation.

VI. References

[1] J. Weiss, Z. Cendes, "Efficient finite element solution of multipath eddy current problems," *IEEE Trans. Magn.*, vol. 18, no. 6, pp. 1710-1712, Nov. 1982.

[2] Seung-Myen Lee, Se-Hee Lee, Hong-Soon Choi and Il-Han Park, "Reduced modeling of eddy current-driven electromechanical system using conductor segmentation and circuit parameters extracted by FEA," *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1448 - 1451, May 2005.

[3] F. Charlet, J. F. Carpentier, "Extraction of 3D interconnect impedances using edge elements without gauge condition," *International Conference on Simulation of Semiconductor Processes and Devices*, 2002. pp. 143 - 146.

[4] R. Asensi, R. Prieto, J. A. Cobos and J. Uceda, "Modeling high-frequency multiwinding magnetic components using finite-element analysis," *IEEE Trans. Magn.*, vol. 43, no. 10, pp. 3840-3850, Oct. 2007.