Modeling of Trichel Pulses in the Negative Corona on a Line-to-plane Geometry

Han Yin, Bo Zhang, Jinliang He, *Fellow*, *IEEE*, and Chijie Zhuang
Department of Electrical Engineering, Tsinghua University, Beijing 100084, China
Email: hanyin1986@gmail.com

***Abstract*—In this paper, a 2-D time domain method is proposed to simulate the Trichel pulses in the negative corona discharge on a line-to-plane geometry. Three ionic species in the ionization layer, including electrons, positive ions and negative ions, are taken into consider. The finite element method (FEM) is used to solve the Poisson's equation, while the finite volume method (FVM) is used to solve the charge transport equations. Trichel pulses of a line-to-plane geometry are analyzed and compared with the measured results of a designed experiment in the laboratory. Well agreement is obtained between the calculated results and the measured ones, verifying the validity of the proposed scheme.**

***Index Terms*—Corona, current measurement, finite volume method (FVM), time domain analysis.**

I. Introduction

Corona discharge on the high voltage direct current (HVDC) transmission line causes many electromagnetic related environmental problems, which are increasing public concern nowadays. Numerical study of corona discharge can help to understand the principle of these electromagnetic issues. In the numerical simulation of corona discharge on the overhead transmission line, the electrons are usually ignored and the steady state of the positive and negative ions, which is called as ion flow field, is solved to obtain the quasi-static characteristics, such as electric field and ion current density [1], [2]. However, few studies on the transient process of corona, especially the nanosecond pulse discharge, which is very important to analyze the high-frequency characteristics, such as radio interference and audible noise, are carried out.

In this paper, a time domain finite volume based approach is proposed to simulate the Trichel pulses in the negative corona discharge. The simulated results are compared with the measured results of a reduced-scale experiment, verifying the validity of the proposed approach.

II. Trichel Pulse Modeling

A. Governing equations

The model is constructed in two-dimensional domain and the corona discharge can be mathematically described by the following equations [3].

The electric field satisfies the Poisson's equation

$$\nabla^2 \Phi = -\frac{e\left(N_p - N_n - N_e\right)}{\varepsilon_0}. \qquad (1)$$

The charge densities satisfy the charge transport equations

$$\frac{\partial N_e}{\partial t} + \nabla\cdot\left(N_e \boldsymbol{v}_e - D_e \nabla N_e\right) = \left(\alpha - \eta\right) N_e \left|\boldsymbol{v}_e\right| - \beta N_e N_p \qquad (2)$$

$$\frac{\partial N_p}{\partial t} + \nabla\cdot\left(N_p \boldsymbol{v}_p\right) = \alpha N_e \left|\boldsymbol{v}_e\right| - \beta N_p \left(N_e + N_n\right) \qquad (3)$$

$$\frac{\partial N_n}{\partial t} + \nabla\cdot\left(N_n \boldsymbol{v}_n\right) = \eta N_e \left|\boldsymbol{v}_e\right| - \beta N_n N_p \qquad (4)$$

where, Φ is the electric potential; N_e, N_p and N_n are the densities of electrons, positive ions and negative ions, respectively; e is the charge of an electron; ε_0 is the permittivity of free space; $\boldsymbol{v}_e$, $\boldsymbol{v}_p$ and $\boldsymbol{v}_n$ are the velocity vectors of three ionic species, which can be calculated as

$$\begin{cases} \boldsymbol{v}_e = -\dfrac{3.2376}{\left|\boldsymbol{E}\right|^{0.285}}\boldsymbol{E}, & \text{for } \left|\boldsymbol{E}\right| \le 7.6\times 10^6 \text{ V/m} \\ \boldsymbol{v}_e = -\dfrac{14.5958}{\left|\boldsymbol{E}\right|^{0.38}}\boldsymbol{E}, & \text{for } \left|\boldsymbol{E}\right| > 7.6\times 10^6 \text{ V/m} \end{cases} \qquad (5)$$

$$\boldsymbol{v}_p = \mu_p \boldsymbol{E}, \quad \boldsymbol{v}_n = -\mu_n \boldsymbol{E} \qquad (6)$$

where, $\boldsymbol{E}$ is the electric field vector; μ_p and μ_n are the mobility of positive and negative ions [4]. α, η and β are the ionization, attachment, and recombination coefficients, respectively; D_e is the diffusion coefficient of electrons.

B. Process overview

The calculation geometry is shown in Fig. 1. The simulation of the nanosecond corona discharge is too difficult if the total region of the line-to-plane geometry is considered. In order to simplify the simulation, an artificial boundary is selected around the conductor. The calculation region inside the artificial boundary, of which the radius is 40 times that of the conductor, contains the ionization layer of the conductor. The potential on the artificial boundary is calculated by the charge simulation method (CSM) and the influence of the ground is taken into consider.

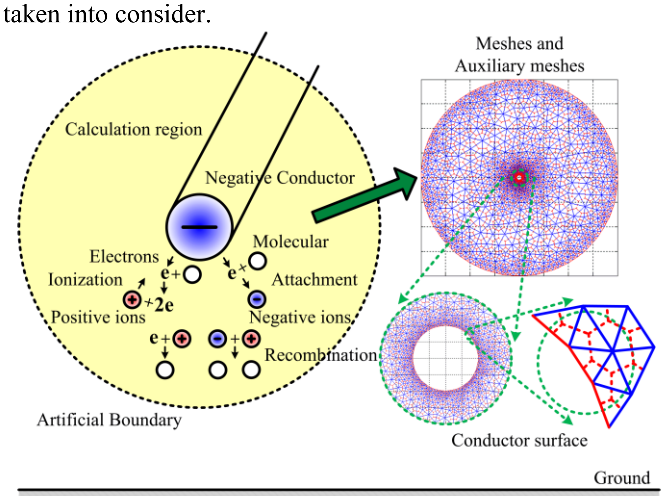

Fig. 1 Calculation geometry and meshes.

In each time step, the Poisson's equation is solved by the finite element method (FEM) to obtain the electric potential and the typical triangular meshes are used, as shown in Fig. 1. Then, the charge transport equations are solved by the finite volume method (FVM) to obtain the charge densities of three ionic species and the auxiliary meshes, which are constructed by connecting the centers of each triangle, are introduced.

C. Solution of the Poisson's equation

The electric potential Φ in each time step is solved by FEM. In order to solve (1), the following function is introduced.

$$R_i = \iint_i \left[\left(\frac{\partial \Phi}{\partial x} \right)^2 + \left(\frac{\partial \Phi}{\partial y} \right)^2 - \frac{2e}{\varepsilon_0} \left(N_p - N_n - N_e \right) \Phi \right] \mathrm{d}x\mathrm{d}y \quad (7)$$

where the subscript i represents the ith element. Then, solving (1) can be transferred to minimizing R_i. Analyzing the derivative of R_i in all elements, a linear equation can be obtained. Combining the equation with the potential boundary condition, the electric potential can be solved.

D. Solution of the charge transport equations

The charge transport equations describe the ionization, attachment and recombination processes in the corona. First, the equation of electrons can be solved independently. Integrating the equation of electrons in the auxiliary mesh around the ith node and discretizing the equation in time domain, the following equation can be obtained.

$$\left[1 - \frac{(\alpha - \eta)\Delta t \left| \boldsymbol{v}_{e,i} \right|}{2} \right] N_{e,i}(t+\Delta t) + \sum_{x=1}^{K} \boldsymbol{F}_{ix} \cdot \boldsymbol{v}_{e,ix} N_{e,ix}(t+\Delta t)$$
$$- \sum_{x=1}^{K} D_e \boldsymbol{F}_{ix} \cdot \nabla \left(N_{e,ix}(t+\Delta t) \right) = \left[1 + \frac{(\alpha - \eta)\Delta t \left| \boldsymbol{v}_{e,i} \right|}{2} \right] N_{e,i}(t)$$
$$- \sum_{x=1}^{K} \boldsymbol{F}_{ix} \cdot \boldsymbol{v}_{e,ix} N_{e,ix}(t) + \sum_{x=1}^{K} D_e \boldsymbol{F}_{ix} \cdot \nabla \left(N_{e,ix}(t) \right) - \beta N_{e,i}(t) N_{p,i}(t) \quad (8)$$

where, t is the time of the current step; Δt is the time step; the subscript i represents the ith node; ix represents the auxiliary edges around the ith node; K is the number of the auxiliary edges; $\boldsymbol{F}_{ix}$ is the related vector, which can be calculated as

$$\boldsymbol{F}_{ix} = \frac{L_{ix} \Delta t}{2 S_i} \boldsymbol{n}_{ix} \quad (9)$$

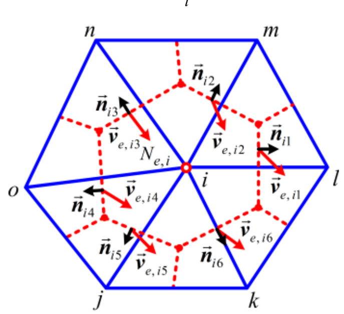

Fig. 2. Calculation of the electron densities by upstream FVM.

where, L_{ix} is the length of the auxiliary edge; $\boldsymbol{n}_{ix}$ is the unit outward normal vector of the auxiliary edge; S_i is the area of the auxiliary mesh around the ith node, as shown in Fig. 2. The densities of electrons on the auxiliary edge are calculated by the upstream method and the gradients of the electron densities are expressed by the one order shape function [5].

After obtaining the densities of electrons, the equations of positive and negative ions can be discretized and solved similarly. In order to accelerate the scheme, the variable time step method is used.

III. VALIDATION

The simulated results are compared with the measured results of a designed experiment in the laboratory. The height of the conductor is 60 cm and the radius of the conductor is 1 mm. The corona current on the conductor is measured by a designed high-frequency current acquisition system, as shown in Fig. 3. When the voltage is -35 kV, the simulated and measured Trichel pulses are shown in Fig. 4. The simulated values match the measured ones well, verifying the scheme.

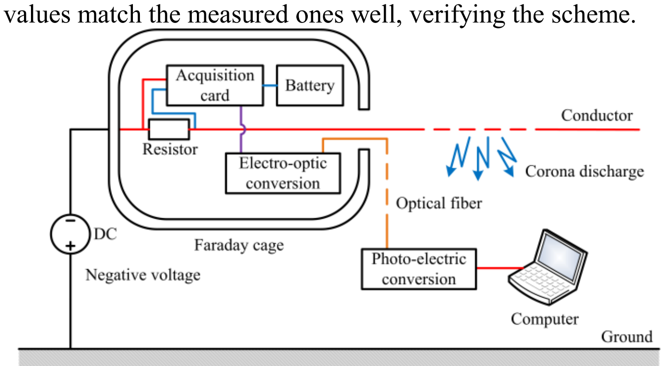

Fig. 3 Measurement platform of the Trichel pulses.

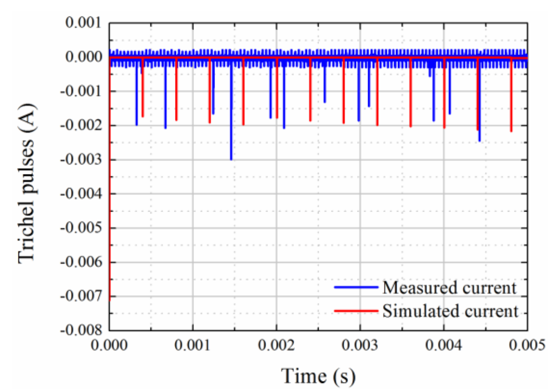

Fig. 4 Comparison of the simulated and measured Trichel pulses.

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