Domain decomposition method used in reducing error near boundaries based on combined RBF collocation

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***Abstract*—Radial basis function (RBF) collocation method used in numerical approximation has some problems. One of them is error near boundaries. When boundary conditions make the numerical approximation function discontinuous, the radial basis function collocation method cannot provide high accuracy solutions. In this work, we use domain decomposition method which can overcome Gibbs phenomenon in computational domain to reduce approximation error near boundaries. In order to implement this method, we extend the computational domain based on the Maxwell equations, so that the boundary problems change into the two sub-domains problems. Two-dimensional hollow potential box is computed to show the efficiency of the proposed method.**

***Index Terms*—accuracy, boundary conditions, approximation error, radial basis function (RBF)**

I. Introduction

It is well know that the radial basis function collocation method has been used in numerical calculation of electromagnetic fields in recent years [1]. The method is one of mesh-less or mesh-free methods which form a new class of numerical techniques to overcome the limitations imposed by traditional mesh structured methods[2], [3]. The radial basis function collocation method as a truly mesh-free method uses a set of nodes instead of traditional mesh elements in computational domain. As we known that this method still has some problems such as Gibbs phenomenon [4] and error near boundaries [5]. These two problems are both because of the global or high order approximations used in the jump discontinuity domain. A lot of work to enhance the accuracy has received very good results. Domain decomposition combined radial basis function collocation method [6], [7] which divides the solving domain into several sub-domains with different media, could be conveniently avoid the discontinuous region, since in each domain there is no discontinuous region, and the notes in the interface between two sub-domains transfer boundary conditions.

As a consequence, we use the same method to reduce the error near boundaries. In general, there are some methods to enhance the accuracy, such as adding polynomial terms, boundary clustering of nodes and virtual boundary, but all of them are all from numerical study not the theoretical study of error near boundaries. Calculation process is not standardized, which make programming more complex.

In numerical calculation of electromagnetic fields, boundary conditions are not only restricted conditions but also a bridge to transfer information of field source from different sub-domains. The governing equations are determined by the practical problems of electromagnetic field and the solving domain is limited in some areas not the whole electromagnetic field. Because of that, we can extend the computational domain based on the Maxwell equations and there is no need to extend a large area, so that we change the boundary problem to two sub-domains problem and use domain decomposition method [6], [7].

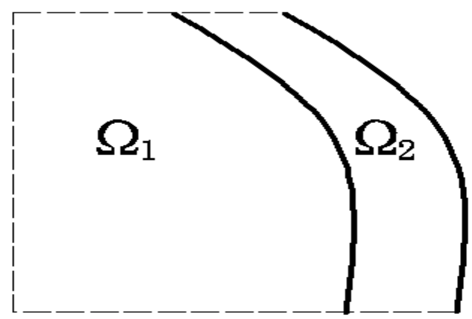

Fig.1.The computational domain

II. Domain decomposition method used in reducing error near boundaries

Fig.1 is the computational domain, Ω_1 is the real solving domain, and Ω_2 is virtual boundary. Using domain decomposition combined radial basis function collocation method, the governing equations are below

$$\begin{cases} L(u_1) = f & in\,\Omega_1 \\ B(u_1) = g & on\ \partial\Omega_1 \cap \partial\Omega \\ u_1 = u_2 & on\ \partial\Omega_1 \cap \partial\Omega_2 \\ \dfrac{\partial u_1}{\partial n} = \dfrac{\partial u_2}{\partial n} & on\ \partial\Omega_1 \cap \partial\Omega_2 \\ L(u_2) = f & in\ \ \Omega_2 \\ B(u_2) = g & on\ \ \partial\Omega_2 \cap \partial\Omega \end{cases} \tag{1}$$

Where Ω is a two sub-regions problem domain and $\partial\Omega$ is the boundary of Ω. $\partial\Omega_1 \cap \partial\Omega_2$ is the boundary between two domains. L is a Laplace operator and B means a boundary operator.

To solve (1), we sequentially set collocation nodes in Ω_1 and Ω_2 , and obtain the RBF approximating form of u_i as

$$u_i = \sum_{n=1}^{N_i} a_n \varphi\left(\left\|x\text{-}x_i\right\|, \ \ c\right) = \boldsymbol{\varphi}^T \mathbf{a} \tag{2}$$

Where $i(i=1,2)$ mean two different domains. x is the coordinate of nodes in the space. $\varphi\left(\left\|x\text{-}x_i\right\|, \ c\right)$ is the RBF centered at node x_i and $\boldsymbol{\varphi}$ is the vector form. $\mathbf{a}$ is the unknown coefficient vector to be determined. c is the shape parameter of RBF. Using proposed method, we can quickly find the result of the problem.

III. NUMERICAL EXAMPLE

To validate the algorithm, potential distribution of two-dimensional square metal box is computed as follows.

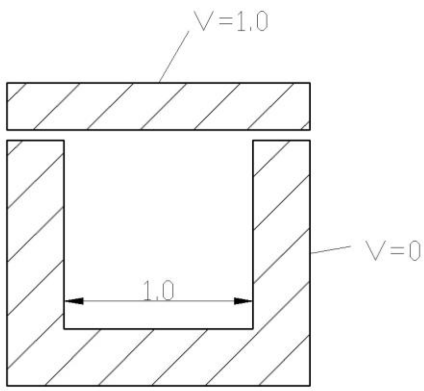

Fig.2. Two-dimensional square metal box

Fig. 2 shows the structure of the metal box, where the electric potential in the cover of the metal box is 1, and other of metal box is 0. Based on the Maxwell equations, we know that the numerical approximations on both sides of metal box may meet the error near boundaries. Fig.3 has proved this point, so we extend the calculation interval. Virtual domain of metal medium is founded, and we update the boundary conditions by equipotentiality in the metal to use the new method to reduce the errors.

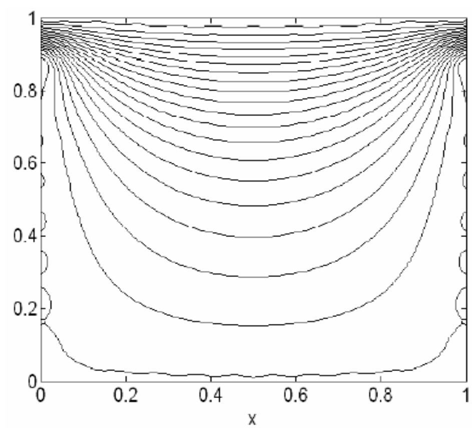

Fig.3. Equipotential lines of the metal box using RBF

Figs. 3and 4 show the result of result with different method. From these figures, we found the jump of equipotential lines on the boundary. That is why we choose it as a numerical example. Compare with the result of ordinary method, the domain decomposition method get a more accurate solution and almost no oscillation. We will compute more numerical example and use different method in the next period of time to ensure our method is useful, which will be appeared in full paper.

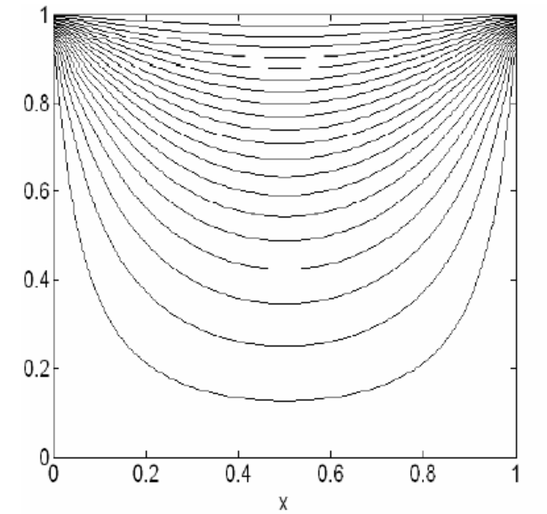

Fig.4. Equipotential lines of the metal box using new method

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