A Novel Approach to Deal with Rotationally Symmetrical Conditions for 3D Eddy Current Field Problems

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***Abstract*—To alleviate the excessive computation burden of numerical methods such as finite element analysis when they are used to solve high frequency three-dimensional (3D) eddy current problems, a novel iterative procedure to deal with rotationally symmetric boundary conditions is proposed in this paper. Numerical results on computations of 3D eddy current fields of a cold crucible are reported to demonstrate the robustness and feasibility as well as the merits of the proposed algorithm for solving practical 3D eddy current problems.**

***Index Terms*—Computational electromagnetics, eddy currents, finite element methods.**

I. Formulations

The vector potential based finite element method is one of the most robust numerical tools for solving 3D eddy current problems. However, the demand for computer resources of this kind of methods will become excessively large when the skin depth of the conducting materials is very small compared to the dimension of the solution domain, since the minimum size of the meshes must be a fraction of the skin depth. To alleviate this problem, a novel iterative procedure to reduce the solution domain in a relatively simple way for solving 3D eddy current problems with rotational symmetries is proposed in this paper.

To facilitate the understanding of the proposed procedure, one considers the 3D eddy current problems of a cold crucible (Fig. 1) with rotational symmetry. A cold crucible is a high frequency induction melting furnace in which the conductive wall is segmented by longitudinal slits and each electrically isolated segment is internally or externally cooled by water [1]. According to the rotational symmetry of the problem, the solution domain can be reduced to one segment region as shown in Fig. 2. Moreover, by using the specific coordinate system of Fig. 2, the symmetrical boundary conditions of the vector potential based finite element models are

$$\frac{\partial \dot{A}_x}{\partial x}|_{S_{12}} = 0, \dot{A}_y|_{S_{12}} = \dot{A}_z|_{S_{12}} = 0 \quad (1)$$

$$\frac{\partial \dot{A}_n}{\partial n}|_{S_{23}} = 0, \dot{A}_\tau|_{S_{23}} = \dot{A}_t|_{S_{23}} = 0 \quad (2)$$

$$|\dot{A}_n|_{S_{23}} = |\dot{A}_x|_{S_{12}} \quad (3)$$

where, n is the outward normal direction of the specified surface, t and τ are the two tangential directions of the specified surface, $\dot{\boldsymbol{Z}}$ means that $\dot{\boldsymbol{Z}}$ is a complex quantity.

Obviously, since surface S_{12} is coincided with plane *yoz*, condition (1) is a natural boundary one, and no additional technique is required to tackle it. For condition 2 and condition 3, due to the arbitrariness in the segments, some specific technologies must be designed to enforce them. Of course, one can use the well known coordinate transformation technique to address this problem. In this paper, a simple, novel and efficient iterative approach is proposed to deal with this kind of Rotational Symmetrical Boundary Conditions (RSBC) and the algorithm will be detailed in the next section.

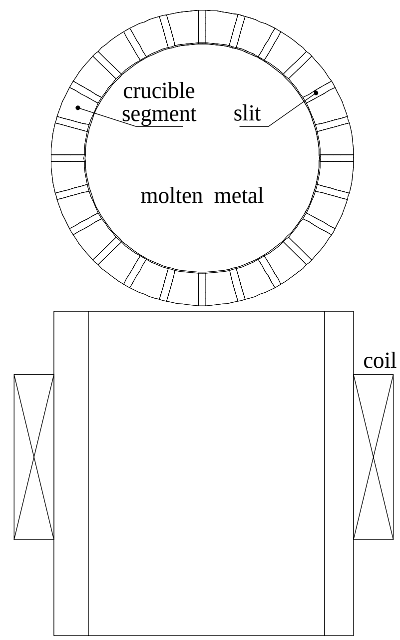

Fig. 1. Schematic diagram of a cold crucible and the rotational symmetry

II. A Novel Approach to Enforce RSBCS

Actually, the proposed approach is iterative in nature and can be described as:

Initialization: Give initial values of the respective vector and scalar potentials $\dot{A}^k, \dot{V}^k$;

Step 1: The potential values at nodes of surface S_{23}, i.e., $\dot{A}|_{S_{23}}^{k'}$, are determined from (4);

Step 2: Resolve the 3D eddy current problem by taking the just determined $\dot{A}|_{S_{23}}^{k'}$ which is known as the first kind of boundary conditions, and let the new solution be designated as $\dot{A}^{k+1}, \dot{V}^{k+1}$;

Step 3: Compare the error of solutions between the two consecutive iteration steps, i.e., $\dot{A}^k, \dot{V}^k$ and $\dot{A}^{k+1}, \dot{V}^{k+1}$.

If the error is within a threshold value predefined by the user, stop the iterative procedure; otherwise, go to Step 2 for the next cycle of iterations.

Moreover,

$$\dot{A}_x |_{S_{23}} = \dot{A}_x |_{S_{12}} \sin(\alpha), \dot{A}_y |_{S_{23}} = -\dot{A}_x |_{S_{12}} \cos(\alpha) \quad (4)$$

where α is the central angle corresponding to one segment region of the crucible as shown in Fig. 2.

Since the ICCG method is used in the proposed method to solve the complex linear equation set, it will be preferable if the value of the predefined precision parameter for terminating the iterative process of ICCG method is set to a large value at the beginning of the iterative process. It will assume a small value towards the end of the iterative process so as to reduce the number of total iterations. In this regard, an adaptive regulation scheme for determining the precision parameter of the ICCG method is proposed as

$$\varepsilon^{k+1} = c \,|\, \dot{A}^k - \dot{A}^{k-1} \,|\, \varepsilon \quad (5)$$

where; ε^{k+1} is the precision parameter used in the $k+1^{th}$ step to stop the ICCG iteration; c is a constant coefficient; $\dot{A}^k, \dot{A}^{k-1}$ are, respectively, the solution of the vector potential at step k and step k-1; ε is a predefined precision parameter.

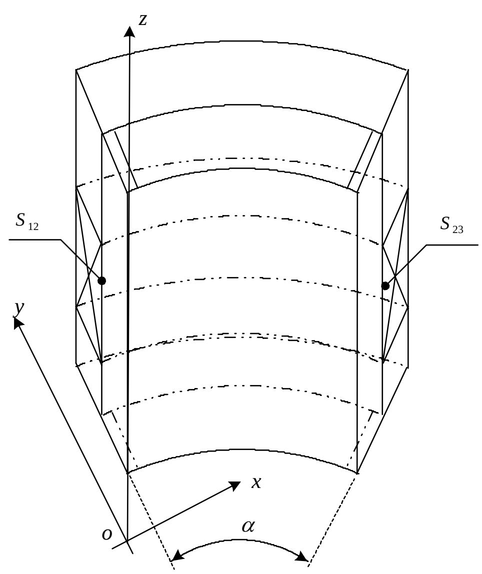

Fig. 2 The reduction of the solution domain (one segment).

III. Numerical Results

To validate the proposed method, a closed slit mold with 48 crucible segments as reported in [2] is analyzed. Fig. 3 illustrates the distribution of the computed magnetic flux density on a predefined path when a 20 kHz harmonic current is applied to the coils. The path is defined as a circle which lies on the surface of the molten metal corresponding to the center position of the coil along the circumferential direction. Fig. 4 demonstrates the distribution of the computed magnetic flux density at points of the slit center which are on the surface of the molten metal along the z-coordinate direction under the same operation condition. The performance comparison of the proposed method with the traditional ones is given in Table 1. It should be pointed out that these numerical results shown good agreement with those of [2]. Thus the numerical results positively confirmed the feasibility and robustness as well as the advantages of the proposed algorithm for solving practical eddy current problems with rotational symmetry conditions.

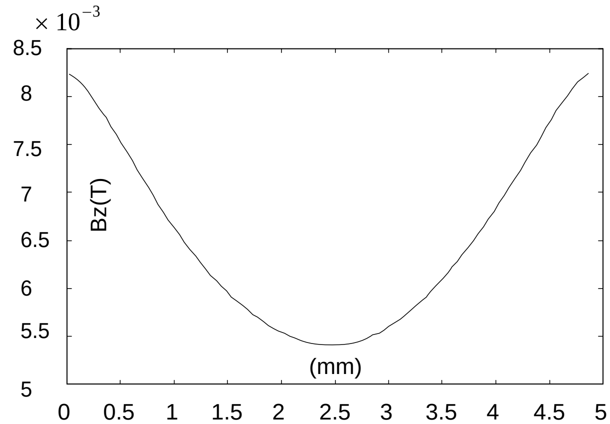

Fig. 3. The distribution of the computed magnetic flux density on the surface of the molten metal along the circumferential directions in the center position of the coil energized with harmonic current at 20 kHz

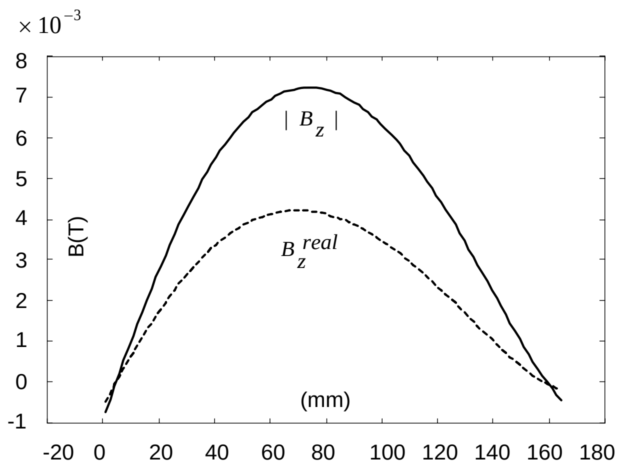

Fig. 4. The distribution of the computed magnetic flux density at points of the slit center which are on the surface of the molten metal along the z-coordinate direction with harmonic current of 20 kHz

TABLE I

PERFORMANCE COMPARISON OF THE PROPOSED AND THE TRADITIONAL APPROACHES

	Nodes	Meshes(Cells)	No. of total DOF
Proposed	16000	18641	54188
Traditional	84000	100846	328600

References

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