Evaluation of Singular Integral Equation in MoM Analysis of Arbitrary Wire Structures

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***Abstract*—This work presents an efficient procedure for evaluating singular integrals arising in the Method of Moment analysis of arbitrary wire structures. The singular kernel is modeled as an elliptical integral of the first kind and evaluated using the Arithmetic Geometric Mean. Numerical integration is applied only for no singular kernels. It is shown that the efficient singularity removal process reduces the condition number of the Method of Moment matrix and the number of base functions required to represent the equivalent current.**

***Index Terms*— Numerical simulation, Moment method and Computational electromagnetic.**

I. Introduction

Wire structures (consisting of elements with diameters much smaller than their lengths) are used in different practical problems of electromagnetic scattering, particularly in antenna applications. Among the most commonly used geometries can be highlighted the circular, spiral, rectangular, elliptical and dipoles. Since the scattering problems involving these geometries are open (problems where the scattered field propagates in all directions without limits), one of the most widely used numerical techniques for their analysis is the Method of Moments (MoM) [1]. Although the MoM efficiency for solving Electric Field Integral Equation (EFIE) in the numerical analysis of the electromagnetic scattering by wire structures was satisfactorily demonstrated [2]-[4], several new algorithms and applications have been investigated in order to make the technique more accurate and efficient. The main factors that motivate the current research are the use of EFIE without approximations, the use of different kinds of base and weight functions, the development of efficient techniques for treating singularities present in EFIE evaluated by MoM and building solutions for analysis of arbitrary geometries beyond the investigation of different practical applications.

In the MoM technique, the choice of base functions, for the representation of the equivalent current, and weight functions is very important for the accuracy and convergence of the numerical analysis. However, sophisticated base or weight functions may lead to complicated integrands and, consequently, singularity removals. To obtain accurate numerical results considerable precautions should be taken in the singularities treatment [5].

In the present work a robust numerical technique is proposed to remove the singularities arising in the EFIE associated with electromagnetic scattering by arbitrary wire structures. For MoM analysis triangular base and weight functions (TF) are used. The TF, normally, ensures a good representation of current physical behavior and provides relatively simple integrands in the MoM analysis. In the present work the TF is defined over two consecutive segments, as illustrated in Fig. 1. The Galerkin method is used in the MoM analysis and, consequently, the weight functions are defined as well. The technique proposed for singularity removal models the singular kernel as an elliptical integral of the first kind and evaluates it using the Arithmetic Geometric Mean (AGM). Gaussian quadrature is applied only to evaluate integrals with no singular kernels.

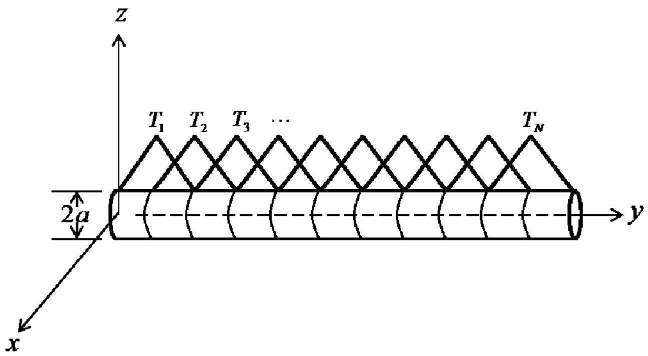

Fig. 1. Distribution of triangular functions

II. Integral Equation Evaluation

The EFIE for arbitrary wire structures is [6]:

$$\hat{n}\times\boldsymbol{E}_t^i(\vec{r}') = \hat{n}\times\frac{j}{\omega\varepsilon}\int_{t'}\begin{bmatrix} k^2\boldsymbol{I}(t')G(\boldsymbol{r}-\boldsymbol{r}')- \\ \nabla'\boldsymbol{I}(t')\nabla'G(\boldsymbol{r}-\boldsymbol{r}')\end{bmatrix}dt', \qquad (1)$$

where $\hat{n}$ is the unit vector normal to the wire direction, t, k is the wave number, $\boldsymbol{E}_t^i$ is tangential component of the electric incident field, $\boldsymbol{I}$ is the surface current in the wire, ε is the electric permittivity of the medium, $\boldsymbol{r}$ and $\boldsymbol{r}'$ represent the observation and source points, respectively and $G(\boldsymbol{r}-\boldsymbol{r}')$ is the Green free-space function.

The MoM solution of (1) using TF as base and weight functions, leads to integrals in the form:

$$\mathrm{I} = \int_{\alpha=-1}^{1}\int_{\alpha'=-1}^{1} a\,b\left(e^{-jkR}/R\right)d\alpha'\,d\alpha, \qquad (2)$$

where R is the distance between source and observation points, a and b may be equal to 1, α or α'. The integrals in (2) have removable singularities whenever the observation point is very close to the source point, i.e., when $R\to 0$.

The technique used in this work to treat these singularities is redefine (2) as an average around the circumference of the source and observation segments:

$$\mathrm{I}=\frac{1}{2\pi}\int_{\gamma}\frac{1}{2\pi}\int_{\gamma'}\left[\int_{\alpha=-1}^{1}\int_{\alpha'=-1}^{1}ab\frac{e^{-jkR_1}}{R_1}d\alpha'd\alpha\right]d\gamma'd\gamma, \tag{3}$$

where γ and γ' are the angles around the circumference of the source and observation segments and R_1 is

$$R_1^2=R^2+\left(a+a'\right)^2-4aa'\cos\left[\left(\gamma-\gamma'\right)/2\right]. \tag{4}$$

Considering the change of variable $\left(\gamma-\gamma'/2\right)\rightarrow\left(\varphi-\pi/2\right)$ (3) can be separated into two parts and rewritten as:

$$\mathrm{I}=\frac{1}{2\pi^2R_2}\int_0^{2\pi}\frac{d\varphi}{\sqrt{1-\sigma^2\sin^2\varphi}}+\frac{1}{8\pi^2}\int_0^{2\pi}\frac{e^{-jkR_1}-1}{R_1}d\left(\gamma'-\gamma\right), \tag{5}$$

where $R_2=\sqrt{R^2+\left(a+a'\right)^2}$.

The second term of (5) does not have singularities and is numerically evaluated by Gaussian quadrature. The first term of (5) has the form of a first kind elliptical integral and presents singularities. In this work this term is evaluated using Arithmetic Geometric Mean according to the algorithm illustrated in Fig. 2 [7]. This procedure leads to highly accurate results even for σ → 1 (Singularity).

<u>VariableS</u>

$a_0=1,\quad b_0=\sqrt{1-\sigma^2}$

$c_0=\left(1-\sigma^2\right)/b_0,\quad d_0=\sigma^2/\left(1-\sigma^2\right)$

$f_0=0,\quad i_0=1/2,\quad s_0=i_0\sqrt{a_0-b_0}$

<u>Repeat</u>

$a_{n+1}=\left(a_n+b_n\right)/2,\quad b_{n+1}=\sqrt{a_nb_n}$

$i_{n+1}=2i_n,\quad s_{n+1}=s_n+i_{n+1}\sqrt{a_{n+1}-b_{n+1}}$

$c_{n+1}=\left(b_{n+1}/4a_{n+1}\right)\left(2+c_n+1/c_n\right)$

$d_{n+1}=\left(c_{n+1}d_n+f_n\right)/\left(1+c_{n+1}\right)$

$f_{n+1}=\left(d_n+f_n\right)/2$

<u>Until</u> $\left|a_{n+1}-b_{n+1}\right|<0.00001$

$\int_0^{2\pi}d\varphi/\sqrt{1-\sigma^2\sin^2\varphi}=\pi/2a_{n+1}$

Fig. 2. AGM algorithm [7]

III. Numerical Results

To evaluate the accuracy of the technique proposed we consider a dipole with a radius of 0.01 λ_0 and a length of 0.47 λ_0, where λ_0 is the vacuum wavelength. The excitation is performed using the Delta Function Excitation Method [6]. The accuracy of the numerical results was verified against the result obtained by software 4NEC2 using the following Mean Relative Error:

$$E_{MR}=\frac{100}{N}\left[\sum_{i=1}^{N}\frac{X^{MoM}\left(j\right)-X^{4NEC2}\left(i\right)}{\max\left(X^{4NEC2}\left(i\right)\right)}\right](\%), \tag{6}$$

where X represents the results obtained by MoM or 4NEC2 and N is the number of segments used to describe the dipole.

In the analysis N varied from 9 to 41, the E_{MR} obtained, for current distribution, is shown in Fig. 3 and the condition number in Fig. 4. In both analyses we consider the situation where the technique to remove the singularities is and is not applied. As can be verified, when this technique is applied the number of base functions required to represent the equivalent current can be reduced once the E_{MR} is always less, independent of the number of segments used. It can also be observed that when the extraction technique is used, the MoM matrix is better conditioned, especially when the number of segments employed is greater.

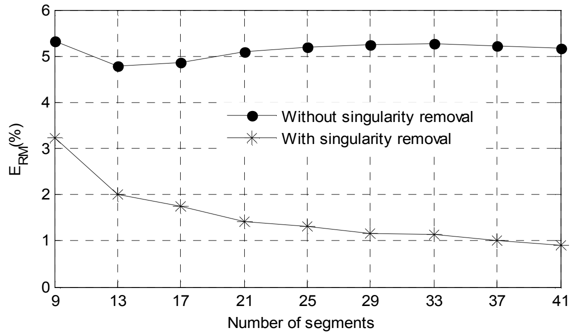

Fig. 3. E_{RM} with and without singularity removal

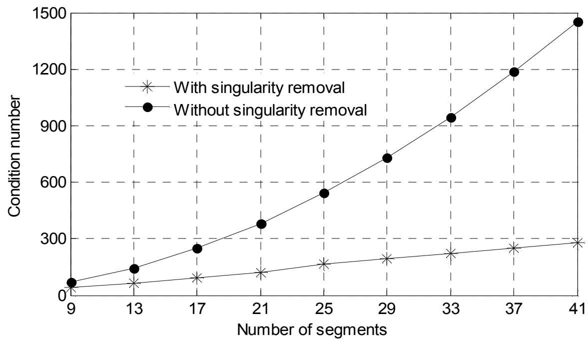

Fig. 4. Condition number with and without singularity removal

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