The Adaptive Cross Approximation Technique for Volume Integral Method Applied to Nonlinear Magnetostatic Problems

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***Abstract*—The volume integral methods are particularly well suited to compute field in the air domain which don't need to be meshed. However, their application leads to solve dense matrix systems. The Adaptive Cross Approximation (ACA) is an algebraic method allowing the compression of these matrices. This paper presents the ACA technique applied for a Volume Integral Method (VIM) in order to solve nonlinear magnetostatic problems.**

***Index Terms*— Adaptive Cross Approximation, acceleration method, volume integral equations, magnetostatics, nonlinear resolution.**

I. Introduction

Integral methods have many advantages over finite elements method such as: far interactions are computed accurately, and especially avoiding the mesh of the air. Their main drawback is the obtaining of a full interaction matrix whose computation time and memory size increase in a quadratic complexity. Many methods have been developed to limit this drawback. The Fast Multipole Method (FMM) [1] is one of the most successful approaches. However this method presents some difficulties in regard to the algorithm parallelization and to the solver preconditioning.

The Adaptive Cross Approximation [2] has been successfully applied for many integral methods such as the Boundary Element Method [3] and the Moment Method [4]. This approach is less intrusive in the source code, allows better solver preconditioning than the FMM and is easily parallelizable.

In this paper, the application of ACA for the Volume Integral Method to solve nonlinear magnetostatic problems is presented. The section II presents the integral formulation using magnetic scalar potential. The section III introduces the resolution of the nonlinear magnetostatic problems. The outlines of the ACA and of the hierarchical matrix methods are described in the section IV. The section V is dedicated to a numerical example.

II. Volume Integral Equation Formulation

Let us consider a ferromagnetic material placed in a static magnetic source field $\mathbf{H_0}$. The magnetic behavior law is defined by:

$$\mathbf{M} = \chi(\mathbf{H})\,\mathbf{H} \quad , \tag{1}$$

where $\mathbf{M}$ is the magnetization, $\mathbf{H}$ the magnetic field and χ the magnetic susceptibility. Let us consider that the material region is simply connected and containing no current sources. The magnetic fields $\mathbf{H}$ and $\mathbf{H_0}$ derive then respectively from the magnetic scalar potential Φ and Φ_0. The volume integral method [5] is used and we can write:

$$\Phi(\mathbf{r}) + \frac{1}{4\pi}\iiint_V \chi(\mathbf{H})\,\nabla\Phi(\mathbf{r}')\cdot\frac{\mathbf{r}-\mathbf{r}'}{\|\mathbf{r}-\mathbf{r}'\|^3}\,dV = \Phi_0(\mathbf{r}) \,, \tag{2}$$

where V is the volume of the material, $\mathbf{r}$ and $\mathbf{r'}$ are respectively the coordinates of computation and integration points.

Only the material region is meshed and the magnetic scalar potential is discretized with first order nodal shape functions. A collocation approach applied on (2) at the N mesh nodes leads to a system of algebraic linear equations:

$$\left([\mathrm{I}] + [\mathrm{A}(\chi)]\right)\mathbf{\Phi} = \mathbf{\Phi}_0 \,, \tag{3}$$

where [I] is the identity matrix and [A] the full interaction matrix.

III. Nonlinear Formulation

This part proposes a modified fixed point method [6] to solve the nonlinear magnetic field problem.

The behavior law (1) is written:

$$\mathbf{M}(\mathbf{H}) = \chi_{FP}\,\mathbf{H} + \mathbf{S}(\mathbf{H}) \quad , \tag{4}$$

where χ_{FP} is the constant slope of the modified point scheme and $\mathbf{S}$ the nonlinear residual. The fixed point can be found by iteratively updating the nonlinear residual. Using the behavior law (4) instead of (1) in the VIM (2), the following equation must be solved at each iteration:

$$\begin{aligned}&\Phi^k(\mathbf{r}) + \frac{1}{4\pi}\iiint_V \chi_{FP}\,\nabla\Phi^k(\mathbf{r}')\cdot\frac{\mathbf{r}-\mathbf{r}'}{\|\mathbf{r}-\mathbf{r}'\|^3}\,dV \\ &= \Phi_0(\mathbf{r}) + \frac{1}{4\pi}\iiint_V \mathbf{S}^k(\mathbf{r}',\mathbf{H}^{k-1})\cdot\frac{\mathbf{r}-\mathbf{r}'}{\|\mathbf{r}-\mathbf{r}'\|^3}\,dV\end{aligned} \quad , \tag{5}$$

where k is the iteration number. Using the previous formulation (2) and (3), the resolution of (5) leads to a system of algebraic linear equations of the form:

$$\left([\mathrm{I}] + [\mathrm{A}(\chi_{FP})]\right)\mathbf{\Phi} = \mathbf{\Phi}_0 + \mathbf{B}(\mathbf{S}^k) \,, \tag{6}$$

where $\mathbf{B}$ is the contribution vector of the nonlinear residual.

The value of the residual S after the iteration k is given by:

$$\mathbf{S}^{k+1} = \mathbf{M}(\mathbf{H}^k) - \chi_{FP}\,\mathbf{H}^k \,,. \tag{7}$$

If the norm of the difference of χ between two iterations is lower than a given criterion, the algorithm is stopped.

IV. The Adaptive Cross Approximation Technique

A. *Outline of the ACA technique*

The ACA technique is based on the approximation of a block matrix by a product of two matrices of smaller sizes. This product approximates the initial matrix by one with a rank much lower but with a sufficient accuracy. If $\mathbf{M}_{m\times n}$ is the considered m×n block matrix, its approximation $\tilde{\mathbf{M}}_{m\times n}$ can be written as :

$$\tilde{\mathbf{M}}_{m\times n} = \mathbf{U}_{m\times p}.\mathbf{V}_{p\times n}, \qquad (8)$$

where $\mathbf{U}_{m\times p}$ and $\mathbf{V}_{p\times n}$ are respectively m×p and p×n matrices. This decomposition is only useful if p < ½ min{m,n}. If the block is sufficiently smooth, the p value can be sufficiently low to lead to a high compression rate. Due to the approximation property (8), the following estimate holds

$$\left\|\mathbf{M}_{m\times n} - \tilde{\mathbf{M}}_{m\times n}\right\|_F \le \varepsilon \left\|\mathbf{M}_{m\times n}\right\|_F, \qquad (9)$$

where $\|.\|_F$ denotes the Frobenius norm of a matrix, and ε is a given criterion. The full ACA algorithm is presented in [2].

The main advantage of ACA decomposition is that the evaluation of the entire matrix is not needed. Indeed, only the knowledge of *p* lines and *p* columns of the matrix is required. Therefore, it does not only decrease the needed memory, but also greatly limits the number of integral computations.

B. *Hierarchical Matrices*

To be efficient, the matrices to compress must involve interactions between distant subspaces where the integration kernel is smooth. To fulfil this criterion, the degrees of freedom in the mesh are renumbered. This task is performed with the help of a partition of the space thanks to an octree. A representation of the total matrix into sub-matrices is then obtained. It is called hierarchical matrix. Each sub-matrix is dealt either as near or as far interaction. In the first case, the sub-matrix is classically evaluated by a full matrix computation. In the second case, the block is compressed with ACA.

For the nonlinear solving, the assembly of vector **B** in (6) is decomposed into a product of the form:

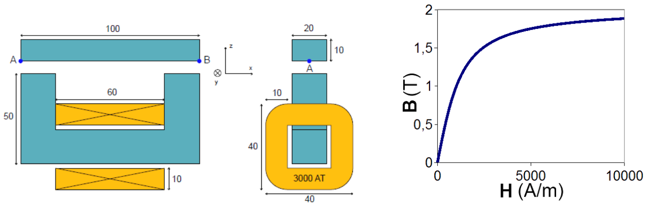
$$\mathbf{B}(\mathbf{S}) = \begin{bmatrix}[B_x] & [B_y] & [B_z]\end{bmatrix}\begin{pmatrix}\mathbf{S_x}\\ \mathbf{S_y}\\ \mathbf{S_z}\end{pmatrix}, \qquad (10)$$

where $[B_x]$, $[B_y]$, $[B_z]$ are assembly matrices of the derivative Green kernel and $\mathbf{S_x}$, $\mathbf{S_y}$, $\mathbf{S_z}$ are the three components of S. $[B_x]$, $[B_y]$, $[B_z]$ are built only once and only depend on the mesh geometry.

The ACA method combined with Hierarchical matrices are used to compute the first order matrix assembly [A] in (4) and the matrices assembly $[B_x]$, $[B_y]$ and $[B_z]$ in (10).

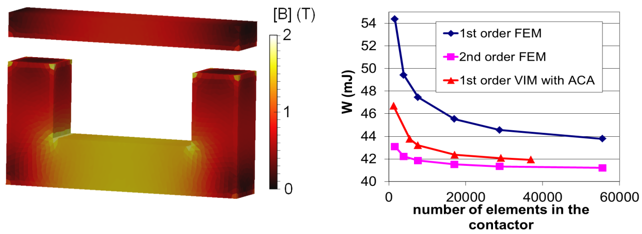
V. Numerical Results

Let us consider the following contactor-like problem. The geometry description and the magnetic behavior law are described in Fig. 1a and 1b. The proposed procedure is applied for a mesh of 37000 elements.



a. Contactor geometry b. Magnetic behavior law

Fig. 1. Description of the contactor-like problem



a. Magnitude of magnetic field B b. Computed energy evolution

Fig. 2. Numerical results on a Intel Core i5 2.2GHz x64 CPU with 4Go RAM

With a 1e^{-4} ACA criterion, only 1.9GB RAM are required for the assembly of the 4 interaction matrices in (6) and (10). Without compression, 8.3GB RAM would be necessary, thus the memory need is decreased by 77% with the proposed procedure. By using the modified fixed point method for (5)-(7) with a χ_{FP} of 650, the nonlinear resolution converges after 158 iterations with 6s for each iteration time. The computed magnetic field is given in Fig. 2a. The FEM is also applied to solve this problem using 1st and 2nd order magnetic scalar potential formulations thanks to the software FLUX®. The evolution of the total magnetic energy of the contactor with respect to the number of mesh elements is given in Fig. 2b. The three results converge to the same value. For the 1st order, VIM provides an accurate energy with relatively few elements.

VI. Conclusion

The ACA compression technique has been applied to a nonlinear magnetostatic problem with a VIM using the magnetic scalar potential. The results are interesting concerning the treatment of fine meshes with a gain of memory.

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