Reducing the cost of mesh–to–mesh data transfer

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***Abstract*—Mesh-to-mesh data transfer has a wide variety of applications ranging from field discretization to coupled system modeling. Up to now, the main drawback of the projection method is its computational cost. This work presents new methods which reduce this cost and avoid the resort to linear systems.**

***Index Terms*—Projection algorithms, Computational efficiency, Computational geometry, Function approximation, Scientific computing, Magnetic Fields.**

I. Introduction

The advanced modeling of electric devices –where coupled phenomena are present– usually requires the development of a specific computer program resorting to different physical models, which is often time consuming and application-oriented thus leading to a narrow scope of use. Along with the increasing power of computational tools, the need for flexible software makes the use of indirectly coupled methods increasingly common. These methods present many advantages such as the the easy replacement of subcodes, the numerical adaptation of meshes, or easy performing remeshing processes. The most substantial point is the ability of using different meshes which are spatially restricted to the absolute useful parts and numerically adapted. The use of projections has been show to not significantly decrease the accuracy [1], and that the efficiency strongly depends on both the coupling scheme and the data transfer.

After a description of the projection methods, new approaches using approximated mesh-to-mesh data transfers will be presented. Numerical applications will allow the measurement of the efficiency of each method in terms of accuracy and computational cost.

II. Field projection method

A. Exact field projection

The basis for data transfer from one mesh to another is the equality between the functions supported by each mesh. Considering two disconnected meshes denoted by $\mathcal{T}_m$ and $\mathcal{T}_s$, the aim is to solve:

$$u_m - u_s = 0, \tag{1}$$

at each point of the domain common to the two triangulations, and denoted Ω_c[1]. The subscript m refers to the "master" mesh for which the function u_m is known, whereas s refers to the "slave" mesh for which u_s has to be determined.

Functions u_m and u_s are discretized using Whitney elements and can be expressed by the linear combinations: $u_m = \sum w_{m_i} d_{m_i}$ and $u_s = \sum w_{s_i} d_{s_i}$. Functions w_{m_i} and w_{s_i} generically refer to the basis functions of the master and slave meshes. d_{m_i} and d_{s_i} are the associated weights, and d_{m_i} represents the source data while d_{s_i} are unknowns.

[1]This domain may not entirely cover mesh $\mathcal{T}_m$ or $\mathcal{T}_s$

Because of the discretization, equation (1) cannot be satisfied at each point of the domain. We then use a weak form of this equation defined as:

$$\forall \psi \in U \quad \langle u_m - u_s, \psi \rangle_U = 0. \tag{2}$$

Depending on the type of finite element used, U successively refers to the $\mathcal{L}^2$, $\mathcal{H}^{grad}$, $\mathcal{H}^{div}$, or $\mathcal{H}^{curl}$ functional spaces. In accordance with the space U, one of the following dot-products can be used:

$$(u, v) \in \mathcal{L}^2 \quad \langle u, v \rangle_U = \int_{\Omega_c} u \cdot v, \tag{3a}$$

$$(u, v) \in \mathcal{H}^D \quad \langle u, v \rangle_U = \int_{\Omega_c} u \cdot v + D(u) \cdot D(v), \tag{3b}$$

where D successively represents grad, curl, or div when respectively node, edge, or face-based finite elements are used. In equation (2), ψ is successively replaced by the basis functions associated to the slave mesh, leading to the conform Galerkin method. It ensures the slave function u_s to be the closest one –according to the norm defined by the considered inner product– to the function u_m. The method leads to the linear system: $[M][U_s] = [N][U_m]$ where $[U_m]$ and $[U_s]$ are the respective arrays of degrees of freedom (DoF) of the functions u_m and u_s. As an example, for the $\mathcal{L}^2$ space, the entries for the two mass matrices $[M]$ and $[N]$ are:

$$M_{ij} = \int_{\Omega_c} w_{s_i} \cdot w_{s_j} \quad \text{and} \quad N_{ij} = \int_{\Omega_c} w_{s_i} \cdot w_{m_j} . \tag{4}$$

The evaluation of the sum $\int w_{s_i} \cdot w_{m_j}$ requires the determination of the elementary volumes created by the intersection of the two meshes. This stage appears as a major drawback of this method. Despite the existence of efficient intersection algorithms [2], this method is time consuming. An interesting way to solve this probelm is the use of approximated values of N_{ij}.

B. Approximated field projection

Based on an efficient point location algorithm, a good approximation of the sum $\int w_{s_i} \cdot w_{m_j}$ can be obtained using a high number of Gauss points. The function w_{m_j} is then evaluated using the position of the considered Gauss point in the master mesh. This is competitive in terms of computational time, and results show (Table I) that the accuracy is not significantly decreased. However, the overall accuracy is reduced when the function u_m has local variations. The use of a fixed quadrature rule does not take into account these variations. Adaptive integration schemes [3], [4] is a way solve this problem as the density of Gauss points is automatically increased when u_m varies. This technique was used in this study and has revealed particularly efficient if u_m has strong variations. If the numerical quadrature does not match the required accuracy, the mesh is virtually refined and quadrature points are added.

C. Whitney forms interpolation

Another major drawback of the projection method is the resort to a linear system. It imposes the storage of huge sparse matrices, in addition with the resolution of this system. We propose a new method that provides the DoF of the function avoiding the resort to sparse matrices and linear system resolutions. This technique is based on the definition of Whitney forms: the weights of the elements are respectively the values at nodes, the circulation along edges, the fluxes through faces, and the inverses of the volumes. For example, the weight d_{s_i} of the i$^{\text{th}}$ edge of the slave mesh is:

$$d_{s_i} = \int_{\gamma_{s_i}} \sum_{j=1}^{N_m^{edges}} d_{m_j}\, w_{m_j} \cdot d\gamma_{s_i}\,, \tag{5}$$

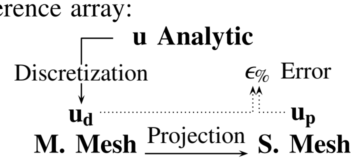
where γ_{s_i} is the path defined by the edge, and $d\gamma_{s_i}$ is the infinitesimal vector directed along the edge. This technique is low memory consuming and is particularly efficient as no linear system is solved. Moreover, for a given order of function, the computation of circulations or fluxes requires a lower number of Gauss points than the thoses required for volumic integrals. As it was done in II-B, the use of adaptive integration schemes is also possible.

III. Numerical example

A. Description

In order to evaluate the global efficiency of these methods (II-B, II-C) in terms of accuracy and computational cost, an analytic case is studied. The considered meshes are based on a unitary cube, and are composed of 1.7 10^6 elements for the master one and 7.1 10^5 elements for the slave one. First, the discretization of the function $u = r^3 sin(\theta)\mathbf{u}_\phi$ is performed through the master mesh thereby producing a reference DoF array. $\mathbf{u}_\phi$ is the spherical coordinates orthoradial unit vector. The expression of u is chosen in order to be general enough for the test. Then, the discrete counterpart of u, stemming from the reference array, is transferred through the second (slave) mesh. The error is computed making the difference between the values of the projected function u_p and the values of the function defined by the reference array:

$$\epsilon_{\%} = \frac{\int \|u_p - u_d\|_D}{\int \|u_d\|_D} \tag{6}$$



where u_d is the discrete reference function and u_p the projected one. These functions are supported by the two different meshes.

B. Numerical results

For the three methods, edge elements and the $\mathcal{L}^2$ functional space have been considered for numerical applications. 45 and 30 Gauss points have been used respectively in the case of

Method	$\epsilon_{\%}$ (%)	Time (s)
Exact field projection	0.45	182
Approximated field projection	0.45	27
Whitney forms interpolation	0.47	6

Table I: Projection error ($\epsilon_{\%}$) for edge elements.

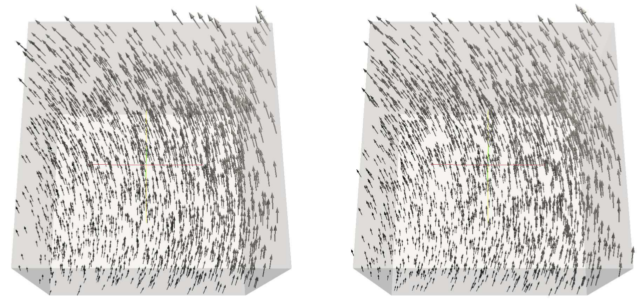


Figure 1: **Right:** Plot of the reference field (master mesh), **left:** transferred field using circulation along edges (slave mesh).

approximated projections and for interpolations along each edge. The three methods widely use loop with independent procedures for which an OpenMP parallelization is close to the optimal speedup. Therefore meshes have to be large enough to obtain substantial acceleration. Figure 1 is a plot of the reference field and the transferred one. Table I presents values of the accuracy and computational time for each method. Computation times are given in for the equivalent sequential computer code, showing the enhancement induced by the approximated projection and the circulation along edges.

IV. Conclusion

Data transfer methods are of great interest to model coupled problems with ease. Previous work [1] has shown that it can be used to model complex systems. The present work focuses on the numerical efficiency enhancement of the data transfer in itself. In a first method, the use of approximated values of the term N_{ij} greatly improves the computation cost, while the accuracy is not significantly decreased. The second method based on the definition of Whitney forms not only improves the computation time but also avoids the resort to linear system and matrix storage.

References

[1] A. A. Journeaux, F. Bouillault, and J. Y. Roger. Multiphysic problems computation using numerically adapted meshes: application to magneto-thermo-mechanical systems. *The European Physical Journal - Applied Physics*, Accepted for publication.

[2] N. Nemitz, O. Moreau, and Y. Ould-Rouis. Magneto-thermal coupling: A conservative-based method for scalar field projection. In *18th Conference on the Computation of Electromagnetic Fields (Compumag 11), Sydney, Australia, Jul 12-15, 2011,*, volume 18, july 2011.

[3] Hashmi Ben Dhia and Guillaume Rateau. The arlequin method as a flexible engineering design tool. *International Journal for Numerical Methods in Engineering*, 62(11):1442–1462, 2005.

[4] A. Nicolet, A. Genon, W. Legros, M. Umé, and F. Delincé. An optimal adaptative numerical integration method. *Albuquerque, USA, Boundary Element Technology VII*, pages 919–933, 1992.

[5] G. Parent, P. Dular, F. Piriou, and A. Abakar. Accurate projection method of source quantities in coupled finite-element problems. *Magnetics, IEEE Transactions on*, 45(3):1132 –1135, march 2009.