An iterative algorithm for the fast analysis of anisotropic magnetic shields

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Abstract—**In this paper the optimal convergence condition of a novel iterative numerical algorithm for the fast analysis of anisotropic magnetic shields is analyzed. This algorithm is based on an iterative scheme by which the convolution properties of the Discrete Fourier Transform are exploited. The theoretical case of a spheroidal magnetic particle is analyzed in order to simply introduce the optimal convergence condition of the proposed iterative algorithm. An example regarding the analysis of a magnetic shield is finally presented.**

Index Terms—**magnetic shielding, numerical methods, FFT.**

I. Introduction

Magnetic field $\mathbf{H}$ inside a magnetic shield can be expressed as

$$\mathbf{H} = \mathbf{H}_s + \mathbf{H}_m, \tag{1}$$

where $\mathbf{H}_s$ is the applied magnetic field generated by the external sources and $\mathbf{H}_m$ is the magnetostatic field contributed by the magnetization $\mathbf{M}$ inside the magnetized material. This field is related to the magnetic induction $\mathbf{B}$ by the expression $\mathbf{B}/\mu_o = \mathbf{H}_s + \mathbf{H}_m + \mathbf{M}$. The numerical computation of the magnetostatic field term $\mathbf{H}_m$ can be obtained, following [1], by discretizing the volume into rectangular blocks and representing magnetization by a discrete distribution of fields $\mathbf{M}_i$ at points $\mathbf{r}_i$. Here, $\mathbf{M}_i$ is the corresponding averaged value on the ith element. The magnetostatic field at $\mathbf{r}_i$ is then

$$\mathbf{H}_{mi} = -\sum_j N(\mathbf{r}_i - \mathbf{r}_j)\mathbf{M}_j \tag{2}$$

where $\mathbf{N}$ is the demagnetizing tensor [1] and $\mathbf{H}_{mi}$ is the averaged magnetostatic field related to the ith element. A similar approach can be found in [2], [3]. As the effectiveness of magnetic shields is based on the high permeability of the soft magnetic materials, we assume magnetic fields that are not too high and static or very low frequency of the applied fields. In such conditions the constitutive relation between variables $\mathbf{M}$ and $\mathbf{H}$ can be considered linear and eventually written as $\mathbf{H} = \mathbf{M}/\chi$, the parameter χ being the magnetic susceptibility. By using this relation in (1) and replacing $\mathbf{H}_m$ in (2) the following equation is obtained:

$$\mathbf{H}_{si} = \mathbf{M}_i/\chi + \sum_j N(\mathbf{r}_i - \mathbf{r}_j)\mathbf{M}_j, \tag{3}$$

where $\mathbf{H}_{si}$ is the applied field averaged on the ith element. Solution of Eq. (3) provides magnetizations $\mathbf{M}_i$ and the whole magnetic field can be eventually calculated at each point inside the shield by using Eq. (2) for the magnetostatic field and

superposing it to applied field in Eq. (1). The computation of magnetic field at any external point can be also obtained for a known $\mathbf{M}$ distribution.

In this paper the novel iterative technique proposed in [4] is applied to anistropic materials and optimal convergence conditions are found with reference to the instructive case a a single rectangular shield under a known external field.

II. Theoretical background

The iterative scheme presented in [4] is briefly summarized in this section. The solution of Eq. 3 can be obtained by the following iterative scheme:

$$\mathbf{M}_i^{k+1} = \mathbf{M}_i^k + \alpha \cdot [\mathbf{H}_{si} - (\mathbf{M}_i^k/\chi - \mathbf{H}_{mi}^k)], \tag{4}$$

where i^{th} index refers to a rectangular block element, $\mathbf{M}_i^k$ is the magnetization distribution at the kth iteration, $\mathbf{H}_{mi}^k$ is the magnetostatic field associated to magnetization $\mathbf{M}_i^k$, the term in the square brackets is the error $\mathbf{e}_i^k$, and α is a relaxation coefficient.

Magnetostatic field at the kth iteration is computed by the formula:

$$\mathbf{H}_{mi}^k = -\sum_j N(\mathbf{r}_i - \mathbf{r}_j)\mathbf{M}_j^k. \tag{5}$$

Here, the convolution product of (5) can be computed very fast by using the FFT algorithm, whose complexity is $O(n \log n)$.

The magnetization distribution is iteratively updated using (5) and (4) until the error norm $\|\mathbf{e}_i^k\|_2$ becomes sufficiently small.

III. Convergence analysys

The analysis of the convergence conditions of the iterative scheme Eq. (4) is based on the Banach Contraction Mapping theorem. Let Γ the functional operator, that in the iterative scheme (4), maps the discretized magnetization distribution $\mathbf{x}_i \in X$ into the new (iterated) value $\Gamma(\mathbf{x}_i)$:

$$\Gamma(\mathbf{x}_i) = \mathbf{x}_i + \alpha \cdot [\mathbf{H}_{si} - \mathbf{x}_i/\chi - \sum_j N(\mathbf{r}_i - \mathbf{r}_j)\mathbf{x}_j] \tag{6}$$

Following this theorem, the iteration (6) converges to its unique Fixed Point $\mathbf{M}_i^*$, i.e. $\mathbf{M}_i^* = \Gamma(\mathbf{M}_i^*)$, if there exists a number $0 < \lambda < 1$ such that for each $\mathbf{x}_i, \mathbf{y}_i \in X$

$$\|\Gamma(\mathbf{x}_i) - \Gamma(\mathbf{y}_i)\|_2 \leq \lambda \cdot \|\mathbf{x}_i - \mathbf{y}_i\|_2 \tag{7}$$

By exploiting Eqs. (6) and (7) we find the convergence condition

$$\lambda(\alpha) = \|(1 - \alpha/\chi) \cdot \delta(i - j) - \alpha \cdot N(\mathbf{r}_i - \mathbf{r}_j)\|_2 < 1, \quad (8)$$

where δ is the Kronecker symbol. Given the parameter α, the value of λ can be calculated numerically from Eq. (8). If $\lambda < 1$ the iterative scheme is convergent. Moreover, the smaller is the value of λ, the faster the convergence of the procedure.

IV. OPTIMAL CONVERGENCE CONDITION

So far a single relaxation coefficient has been employed. The consequences are apparent if related to the instructive case of a homogeneous spheroidal magnetized body. In this case, the demagnetizing tensor is a 3x3 diagonal matrix whose diagonal terms are N_{11}, N_{22}, and N_{33}. Here, $N_{22} = N_{33}$. Moreover, let us assume the case of an anisotropic material with susceptivities χ_1, χ_2 and χ_3. Under these assumptions eq. (8) becomes:

$$\lambda = \max\left\{\left|1 - \frac{\alpha}{\chi_1} - \alpha N_{11}\right|, \left|1 - \frac{\alpha}{\chi_2} - \alpha N_{22}\right|, \left|1 - \frac{\alpha}{\chi_3} - \alpha N_{33}\right|\right\} \quad (9)$$

To illustrate graphically the convergence behavior of the method, we assume $\chi_2 = \chi_3$. The three terms inside braces in eq. (9) are shown in Fig. 1. It is apparent that the 1-st component has a different dependence on α and the application of eq. (9) leads to the dotted curve, whose minumum is attained at α^*.
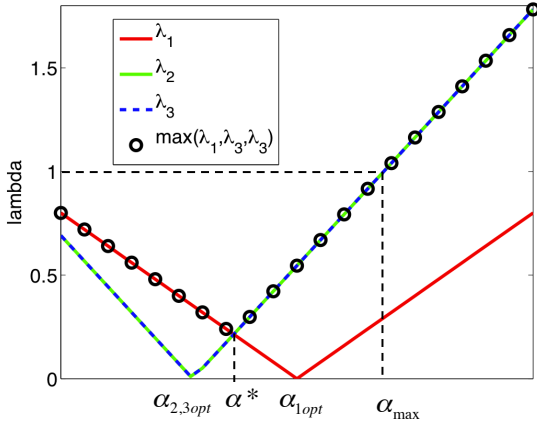

Figure 1. Behavior of the three factors in eq. (9) vs. relaxation factor α. The optimal convergence rate is obtained at $\alpha = \alpha^*$

If different relaxation coefficients are introduced for each axis, a better convergence rate can be obtained. To demonstrate this point, we consider the realistic case of a magnetic anisotropic shield system composed by a square slab with side 2 m, relative permeability 5000 in x direction and 2 in y direction. Here, we assume $\alpha_3 = \alpha_2$, therefore $\lambda = \lambda(\alpha_1, \alpha_2)$. Figure 2 shows the behaviour of the factor λ vs. α_1 and α_2. Here, the global minimum corresponds to the optimal convergence rate. This behaviour is apparent in Fig. 3 where is reported the error norm versus iteration number in the two

cases. The convergence rate is much better then could be obtained by using a single, optimal, relaxation coefficient.
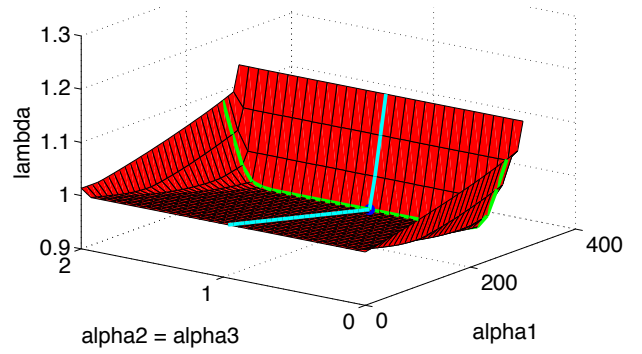

Figure 2. Behavior of factor λ defined in eq. (8), vs. relaxation factors α_1 and α_2. The global minimum corresponds the the optimal values of relaxation factors.
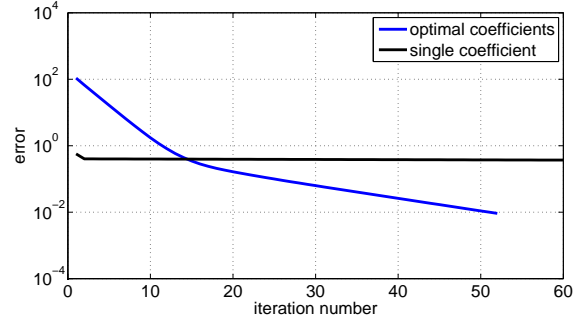

Figure 3. Error norm versus iteration number by using a single relaxation factor (open circle) and different relaxation factors for the two anisotropy directions (open square).

V. CONCLUSIONS

In this paper the optimal convergence conditions of a novel iterative algorithm for the fast analysis of isotropic and anisotropic magnetic shields are analyzed. It is demonstrated that by using different relaxation factors, a faster convergence of the iterative computation can be obtained. In the full paper, practical case study of anistropic magnetic shields and comparison to FEM computations will be presented.

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