To Smooth Vertices in Field Analysis Problems, or Not to Smooth

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***Abstract*—Both conformal mapping via Schwarz-Christoffel formulas and finite element methods can provide accurate results in analyzing two-dimensional fields. In presence of curved boundaries with small radius of curvature, the first are normally constrained to introduce piecewise straight lines, and the original contribution of this work consists of presenting a reliable procedure to smooth several sharp vertices of a polygonal boundary by the formula for rounded corners, and comparing the results with those obtained by replacing sharp corners with piecewise straight lines. Differences are perceived only in close proximity, and this quantitatively explains the similar results obtained from maps and from finite element methods, and provides reliable assessment of the obtainable results.**

***Index Terms*—Static fields, conformal mapping, Schwarz-Christoffel, finite elements.**

I. Introduction

Availability of powerful numerical tools to analyze field problems by means of both conformal mapping via inversion of the Schwarz-Christoffel (SC) formula [1]-[2]-[3] and finite element methods (FEM) [4]-[5]-[6] suggests to discuss an old problem from a methodological point of view, and to make useful comparisons. In fact, the standard SC formula allows us to perform very accurate calculation on domains limited by polygonal boundaries, but this can introduce singularity artifacts where a curved boundary would be more appropriate. Conversely, FEMs require refined discretization procedures [4]-[5] of the analyzed domain, even if very good non-local results are often observed. For instance, a comparison of results was reported in [7], while an initial discussion on local field results has been presented in [8]. The SC formula for rounded corners was early studied [9] but applications have been seldom described, and normally to bend a sole large portion of the boundary, as in [10] (a more general overview of methods is available in [1], chapter 4). In this paper, starting from the same geometry considered in [8], an efficient procedure is described to smooth several sharp vertices to small quasi-circular arcs using the SC formula for rounded corners, and the results are discussed. Comparison is made with results obtained using piecewise straight lines instead of continuous curves, what limits strong field singularities but introduces more singular vertices, and very small differences are observed. This agrees with known FEM performances.

II. Mapping Procedure

The case study geometry is shown in Fig. 1, representing a suitable part of a transformer section, with the low-voltage winding at the lower side, and high-voltage winding and guard-ring in the middle of the figure. In this simulation (applied voltage test) the electrodes are the grounded low-voltage winding and the high-voltage winding connected to the guard-ring; magnetic walls are indicated by dashed lines. A cut was provided between high-voltage winding and guard-ring by means of a pair of conductive sides, to define a simply-connected domain. This was first mapped to the upper SC half plane via numerical inversion of the standard formula (SCNI), and then into a rectangle, were fields and equipotential lines are immediately obtained. From the rectangle, boundary and equipotential lines can be easily remapped to the original or to any new smoothed geometry (plane w), saving global characteristics as capacitances. The original geometry was derived from data used in [8] for FEM analysis, where small circular arcs replaced some right angles, with a radius of 0.004 in the figure units. When rounded corners or short piecewise straight lines of similar lengths are introduced in the geometry, differences among initial and final geometries are hardly perceived at the scale of Fig. 1. So, only local effects will be drawn in the following.

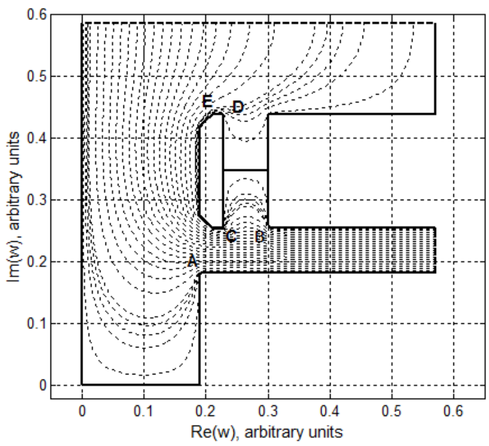

Fig. 1. The analyzed geometry, showing electrodes and equipotential lines.

During SCNI, crowding effects [1] are counteracted by using compound Gauss-Jacoby integration formulas [1][2], and it can be useful to start with a cut of small but finite thickness in the original geometry to avoid double-mapped regions in the final. The equipotential lines are traced from one magnetic walls in the figure to the other by means of a predictor-corrector procedure, and they look near coincident or superimposed in some regions, were potentials differ for a 2% of the applied voltage V. The SC formula for rounded corners leads to quasi circular arcs, whose shape is near independent of the distant vertices of the boundary if the radius is relatively small. This allows us to easily control the arc dimensions, and to avoid optimization procedures if auxiliary vertices are provided at their ends. Accurate integration of the SC formula along the real axis of the intermediate plane easily leads to accurate positioning of all vertices and arcs. The quality can be appreciated in Fig. 2, left and rigth parts, which refer to the vertex regions indicated by A and E in Fig. 1. Four vertices were smoothed to rounded corners, near letters A, B, C and D. The agreement of the behavior of the equipotentials lines with the shape of the neighboring boundary appears both at the left hand, near a smoothed vertex, and at the right, near a non smoothed vertex.

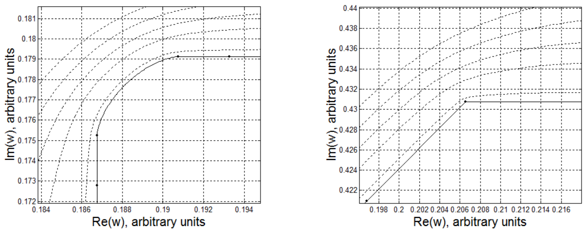

Fig. 2. Details of boudaries and equipotential lines in regions correspondig to those indicated by A (left hand) and E (right hand) in Fig. 1.

III. RESULTS AND DISCUSSION

Being any transformation conformal, electric fields are easily computed at any point both from distances between equipotential lines in close proximity, and from the ratio between a small step along one of them and the step along the corresponding path in the intermediate rectangle. The numerical derivatives are in good agreement, but the second methods leads perhaps to more simple calculations. About 4000 steps are used, staring from the vertical magnetic wall.

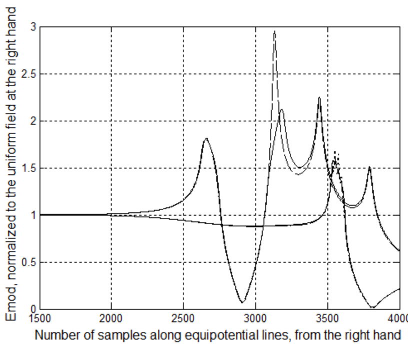

Fig. 3. Electric field amplitudes along two equipotential lines for three geometries: the original (dashed line), that with four rounded corners, and a third, were the rounded corners are replaced by piecewise straight lines (dots).

The fields along the two equipotential lines in Fig. 1 at 0.5% V from the boundary are show in Fig. 3, for three geometries: the original one with piecewise straight lines inscribed in circular arcs at A and B, that in Fig. 2 with four rounded corners, and a new geometry introduced for comparison. This is similar to the original geometry, but the piecewise straight lines are now matched to the rounded corners of the second one. Although this replacement, of course, is not leading to a conformal mapping (the geometrical capacitance varied for about 0.05%), the local field behaviors are suitable to evaluate the influence of the small vertex singularities introduced by the piecewise smoothing. Two sets of three equipotential lines are easily recognized for the two potentials, and a large difference in noted only for the sharp corner in the original geometry near letter C in Fig. 1, rounded or piecewise smoothed in the others. On the contrary, modest differences are perceptible between the fields near the rounded or polygonal boundaries, quantitatively measuring the very local effect of the weak singularities introduced by the five corners of about 18 degrees along the piecewise smoothing. At the same time, this behavior is shown similar to that normally obtained from FEM codes, which neglect very small scale details of boundaries. Moreover, well known rules of thumb are supported, which recommend evaluating fields at some percent voltage distance from sharp edges in boundaries: in fact, more close evaluations are very seldom required.

Further analysis in region A about the rounded corner and about its piecewise version with five vertices shows in Fig. 4 very similar behaviors for equipotential lines at no less than some 5% of the applied voltage V, with small differences in the positions of the local maxima of the field amplitude. So, a conclusion can be drawn as follows: smoothing is often necessary; piecewise smoothing will be normally sufficient.

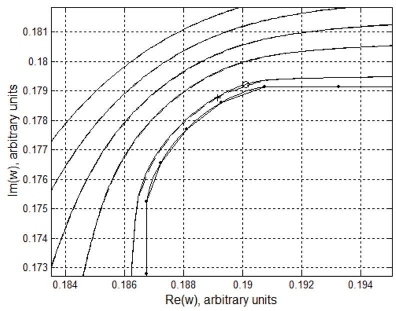

Fig. 4. Equipotential lines and maximum field points for a rounded (solid lines and circle) and a piecewise smooth corner (dashed lines and cross).

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