Comparison of Non-Overlapping Domain Decomposition Methods for the Parallel Solution of Magnetic Field Problems

Dániel Marcsa, Miklós Kuczmann
Széchenyi István University, Department of Automation
Győr, Egyetem tér 1., H-9026, Hungary
marcsad@sze.hu

***Abstract*—The aim of this paper is to give a unified comparison of non-overlapping domain decomposition methods (DDMs) for solving magnetic field problems. The methods under investigation are the Schur complement method and the Lagrange multiplier based Finite Element Tearing and Interconnecting (FETI) method, and their solvers. The performance of these methods has been investigated in detail for two-dimensional magnetic field problems as case studies.**

***Index Terms*—High performance computing, Parallel processing, Finite element methods, Magnetic fields**

I. Introduction

The large scale computations and simulations performed by the finite element method (FEM) [1] often require very long computation time. While limited progress can be reached with improvement of numerical algorithms, a radical time reduction can be made with multiprocessor computation. In order to perform finite element analysis a computer with parallel processors, computations should be distributed across processors [2].

The Schur complement method [2], [3], as sequential algorithm was started to use many decades ago, when computer RAM was extremely small. Nevertheless, nowadays, this method is a very popular parallel domain decomposition technique among engineers [3].

In the last decade, the Finite Element Tearing and Interconnecting (FETI) method [2], [4], [5] has seemed as one of the most powerful and one of the most popular solvers for numerical computation. The FETI requires fewer interprocess communication, than the Schur complement method, while is still offers the same amount of parallelism [2].

This paper presents a parallel approach for the solution of two-dimensional magnetic field problems by parallel finite element method. These problems are benchmarks to show the steps of the DDMs with parallel finite element technique. The comparison focused on the runtime, speedup and numerical performance of solvers of methods in massively parallel environment.

II. Parallel Finite Element Method with Domain Decomposition

The main idea of domain decomposition method is to divide the problem into several sub-domains in which the unknown potentials could be calculated simultaneously, i.e. parallel.

The general form of a linear algebraic problem arising from the discretization of a magnetic field problem defined on the problem can be written as [1], [2]

$$\mathbf{K}\mathbf{a} = \mathbf{b} \tag{1}$$

where $\mathbf{K}$ is the symmetric positive definite matrix, $\mathbf{b}$ on the right hand side of the equations represents the excitation, and $\mathbf{a}$ contains the unknown potentials.

A. Schur Complement Method

After the problem is partitioned into a set of N_S disconnected sub-domains, (1) has been split into N_S particular blocks [2], [3]

$$\begin{bmatrix} \mathbf{K}_{jj} & \mathbf{K}_{j\Gamma_j} \\ \mathbf{K}_{j\Gamma_j} & \mathbf{K}_{\Gamma_j\Gamma_j} \end{bmatrix} \begin{bmatrix} \mathbf{a}_j \\ \mathbf{a}_{\Gamma_j} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_j \\ \mathbf{b}_{\Gamma_j} \end{bmatrix}, \tag{2}$$

where j=1, ..., N_S, $\mathbf{K}_{jj}$ is the symmetric positive definite submatrix of the j^{th} sub-domain, $\mathbf{b}_j$ is the vector of the right hand side defined inside the sub-domain. The sub-matrix $\mathbf{K}_{j\Gamma_j} = \mathbf{K}^{\text{T}}_{\Gamma_j j}$ contains the nodal value of j^{th} sub-domain, which connect to the interface boundary nodes of that region. The $\mathbf{K}_{\Gamma_j\Gamma_j}$ and $\mathbf{b}_{\Gamma_j}$ expresses the coupling of the interface unknowns.

Each sub-domain will be allocated to an independent processor, because $\mathbf{K}_{jj}$, $\mathbf{K}_{j\Gamma_j}$, $\mathbf{K}_{\Gamma_j j}$ and $\mathbf{b}_j$ are independent. Only $\mathbf{K}_{\Gamma_j\Gamma_j}$ and $\mathbf{b}_{\Gamma_j}$ are not independent, these sub-matrices and vectors are stored on the distributed memories.

The assembly of the sub-matrices can be performed parallel by independent processors. However, for the solution of $\mathbf{a}_{\Gamma_j}$ use the sub-matrices from the independent processors. After obtaining the unknowns of interface boundary nodes, it must be sent back to the independent processors to calculate the sub-solutions. The direct solver, the parallel forward-backward algorithm [2] has been used to solve the problems, which will be presented in the full paper.

B. Finite Element Tearing and Interconnecting Method

After mesh partitioning, the FETI method consists in transforming the original problem (1) with the equivalent system of sub-domain equations [2], [4], [5]

$$\mathbf{K}_j\mathbf{a}_j = \mathbf{f}_j - \mathbf{B}_j^{\text{T}}\mathbf{\Lambda}, \tag{3}$$

with the compatibility of the nodal potentials at the sub-domain interface [2], [4], [5]

$$\sum_{j=1}^{N_S} \mathbf{B}_j \mathbf{a}_j = \mathbf{0} \tag{4}$$

where j=1, …, N_S, the number of sub-domains, $\mathbf{K}_j$,$\mathbf{b}_j$ and $\mathbf{a}_j$ are respectively the system matrix, the representation of the excitation and the unknown potentials of j^{th} sub-domain. The vector of Lagrange multipliers $\mathbf{\Lambda}$ introduced for enforcing the constraints (4) on the sub-domain interface, and $\mathbf{B}_j$ is a signed (±) Boolean mapping matrix, which is used to express the compatibility condition at the j^{th} sub-domain interface.

Usually, the partitioned problem may contain $N_f \leq N_S$ floating sub-domains, where matrices $\mathbf{K}_j$ being singular [4]. Because of the floating sub-domain, a robust direct solver or a preconditioned iterative solver is needed to handle the singular matrices. In the full paper we will present the direct solver, and the algorithm of projected conjugate gradient algorithm with lumped preconditioner.

III. Comparison

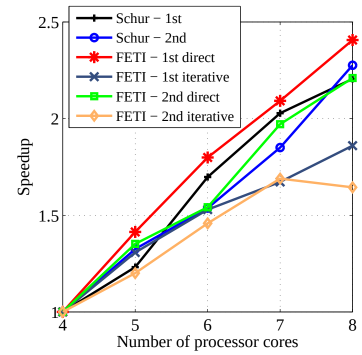
In order to compare the numerical performance of the methods, we have run a number of test cases using a research code that has been developed for that purpose on the Matlab computing environment. Three problems have been used for comparison. The 103828 DoF problem is the induction motor problem. The 135989 and 65245 number of unknowns (DoF) problems are the quarter of single-phase transformer problem, as static magnetic field and eddy current field problems, respectively. The quarter of the transformer contained floating sub-domain, because of the Neumann boundary condition.

Fig. 1 shows the speedup versus the function of the number of the applied processor cores. The 1st and 2nd means first and second order nodal element approximation.

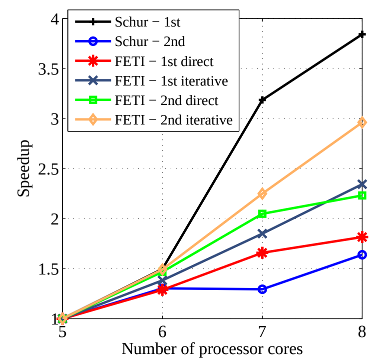
In the first test case (Fig. 1a), the iterative solvers reached less speedup than the direct solvers, because the size of problems per processor core is relatively small. Fig. 1b shows the comparisons through the static field problem, when the problem contains floating sub-domain. In this case, very clearly shows that first order Schur complement method is the best, because the Schur complement method is not as sensitive as the FETI method to the Neumann boundary condition. However, the FETI method is better in the second order case. In case of the eddy current problem (Fig. 1c), the first order Schur complement method has the more than fourfold speedup, but the first order FETI method reached more or less the same speedup at 8 processor core.

IV. Conclusions

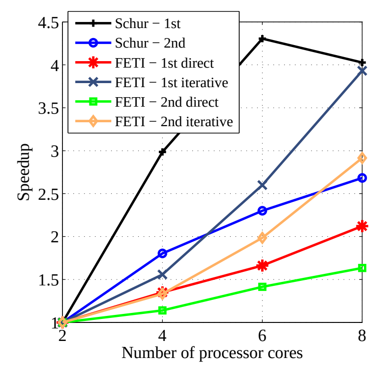
In this paper, we have compared two non-overlapping domain decomposition methods on massively parallel environment. It can be concluded that the Schur complement method is better than the FETI solvers for the first order problems. However, the FETI method with iterative solver is better than Schur complement method for second order problems, because of the less memory requirement of iterative solver.



(a) 103828 DoF problem.



(b) 135989 DoF problem.



(c) 65245 DoF problem.

Figure 1: The speedup of the problems.

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