

Compatible h - p Adaptive Refinement Strategies for Finite Element Electromagnetic Analysis in High Performance Parallel Computing Environments

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Abstract — Compatible h - p refinement strategies are proposed for adaptive finite element modeling, analysis and simulations of electromagnetic applications, within high performance computing environments. The objective of this contribution is to investigate and assess the potential benefits and associated costs of applying advanced adaptive techniques in supercomputing-range parallel processing systems, when the principal adaptive feedback-control algorithms are purpose-built to exploit the innate strengths of the computational environment, and to avoid its primary bottlenecks. A focal point of this study is to examine and quantify the merits of using wide-breadth and deep multilevel h - p refinement trajectory trees, with a set of error-based pruning strategies, for increasing adaptive solution convergence, and for assessing the accuracy and reliability of an evolving finite element model. A foundational set of search strategies is investigated and evaluated computationally, using a series of fundamental benchmark adaptive analyses. The results demonstrate the potential of shallow, wide-breadth search refinement strategies, and support the development and use of a new series of “look-forward” adaptive refinement indicators, that are well-suited to high performance parallel computing platforms.

Index Terms — Computational electromagnetics, finite element method, adaptive mesh refinement, high performance computing.

I. INTRODUCTION

Extending the applicability and reliability of adaptive finite element analysis (AFEA) methods for accurately modeling the increasing complexity of practical electromagnetic devices and systems is an important research area in electrical engineering. A wide variety of h -type and p -type AFEA formulations are in common use and numerous accounts of mounting evidence and support for h - p formulations appear in the published literature. However, truly productive AFEA implementations for parallel processing remain relatively rare, and the reported research on reliable h - p AFEA for practical electromagnetic computational modeling and simulation applications within high performance parallel computing environments is extremely meager [1]-[3].

The fundamental purpose of this contribution is: to clarify the foundational requirements for designing, implementing and using advanced h - p AFEA formulations in large-scale parallel processing environments; to specify and develop the operating principles and strategies required to make them practicable for real-life engineering applications; to test and evaluate realistic implementations within a real-world high performance parallel computing facility. A main focus of this work is to explore the efficacious use of concurrent, competitive and complementary adaptive refinement trajectories [4]. The foundational support elements for the potential and practicality of this approach are:

the strategies used to maintain the multi-version discretization space construct, which keep it both robust and yet manageable; and the techniques and metrics needed to identify, extract and utilize the information available from the secondary refinement trajectories and associated solutions, before they are discarded. Concisely, the essential focus of this contribution addresses the development and use of variable-breadth multilevel refinement trajectory space database searches, with self-adjusting pruning strategies, to stabilize, manage and accelerate the convergence of the h - p AFEA computations, and facilitate the assessment of the validity and accuracy of the evolving finite element model.

II. MULTILEVEL REFINEMENT TRAJECTORY TREES

Using wide-breadth and multilevel discretization trees and techniques, to represent, manage and compare the evolution of parallel h - p AFEA refinement trajectories is straightforward; it was outlined for shallow searches in a previous study [4]. The difference between that research and the present is scale: full-breadth and deep multilevel refinement trajectory modeling for realistic electromagnetic applications is only practicable within high performance parallel computing environments. However, regardless of the processing capacity of the computing facility, the exponential growth of practical refinement trajectory trees will require extensive search tree pruning. Indeed, reliable and effective modeling space data structure pruning strategies will be essential to keep these approaches computationally feasible, even for the largest supercomputing facilities. The underlying goal of this part of the research is to design, implement and use adaptively-tuned pruning methods, to shrewdly limit and focus the potentially unbounded growth of h - p refinement trajectory trees, which can arise with practical electromagnetic analyses.

A spectrum of pruning strategies and criteria, ranging from simple and direct approaches to subtly detailed methodologies, can be developed. Regardless of the formulation adopted, the target is the same: identify the trajectories that are unlikely to evolve towards stable adaptive convergence, and remove them from the growing search space. However, this objective must be attacked conservatively and with judicious care; it is critical to maintain a viable set of seed discretizations at each stage, to facilitate the evolution of the more promising trajectories, i.e., those that are most likely to yield robust and stably convergent analysis models with further refinements. One straightforward, and remarkably effective, scheme is based on a “look-forward” approach that uses the actual rather than anticipated impact of

potential discretization refinements to control model evolution. The scheme is conceptually simple, widely applicable and only practically tenable in high performance parallel environments. In standard AFEA systems, error indicators are used to predict locations in a discretization that appear to need more modeling degrees of freedom (DOF). The look-forward method simply computes the true impact of adding extra DOF, in terms of any error measure of interest. The potential insight and advantages provided by actual errors, rather than uncertain and unreliable a priori estimates, are very attractive for AFEA. However, the related computational expense of obtaining them is equally, or more, substantial. Nevertheless, look-forward refinement error evaluation is feasible, and can be cost-effective and practicable with high performance parallel computing. A broad variety of implementations is possible, ranging from simply assessing the impact of sub-dividing individual elements, up to investigating the available benefits of multilevel, distributed and semi-global h - p refinement scenarios. One basic, but effective, approach is described below, to illustrate this concept. In addition, the key aspects needed to generalize the process to facilitate multilevel search strategies and implementations are briefly summarized.

A. Independent Single-Element Refinement Assessments

A basic yet useful look-forward approach is to compute the actual modeling benefit of refining elements in a discretization individually. Although single-element full-breadth searches are only feasible for smaller discretizations, they give the potential benefit of finding the best single-element discretization update attainable, for each refinement step. While this comprehensive search approach may be intractable for practical problems, the required finite element analyses are independent, and could be computed with suitably large parallel processing environments, in theory. The computational commitment required from each processing unit to handle a typical independent analysis locally dominates the related communications entailed, therefore these processes are well matched to parallel computing realizations. However, while high performance computing facilities grow in scale and throughput continually, it is clear there will never be adequate processing power to foster full-breadth, look-forward approaches, for practical electromagnetics AFEA applications.

This work presupposes that pragmatic look-forward AFEA implementations will be limited by the computing environment used, and the number of processing units available. To delimit the scope further, it will be assumed that it is only permitted to “look-forward” at a fixed amount of the feasible discretization updates at each AFEA refinement step, equal to the number of processing units available. With these restrictions the problem is reduced to selecting the best subset of refinement updates to assemble and compare, at each refinement step. Resolving this restricted search space objective is the main focus of pruning.

To illustrate the aim of the study as directly as possible, for this digest exposition, it is assumed that a typical, a priori error indicator is used to evaluate and order the individual elements, based on their related field solution; and, that the look-forward candidate subset is filled using the worst ranked elements, i.e., those found in most need of added DOF by the error indicator.

While this pruning approach is simple, the underlying concept forms the basis of an effective and efficient class of self-tuning pruning criteria. Specifically, it is feasible to test and monitor a range of competing and complementary error indicators, and then measure their true efficacy, for assessing different parts of the discretization, at each AFEA refinement step, by using the look-forward solutions for empirical feedback and validation.

B. Central Issues of Deep Multilevel Search Approaches

Multilevel discretization refinement trajectory methods can be specified and understood as direct extensions of single-level searches: basically, they are single-level look-forward methods that use search space pruning approaches based on secondary, repeated and recursive, sublevel look-forward analyses. Every viable refinement initiated on one level admits a new spectrum of feasible refinement scenarios, each of which can range from simply including one child of the parent refinement, through to multi-element refinements using many, or all, children. Multi-level search methods are subtle, complex, and computationally intense; their fundamental merit is that they have the ability to effectively identify, isolate and prune ambiguous and dead-end refinement trajectories—even when limited to relatively shallow formulations. Examples will be presented at the conference.

III. ILLUSTRATIVE RESULTS

An illustrative example that represents the studies explored in this research is described below. It is basic, and summarizes a key focus of this work. The performance and processing cost for five variable-breadth, fixed-ply, single-element, search tree schemes are listed in Table I; for converging the stored energy, per unit length, of a thin, straight, infinitely long wire, carrying a direct current in free space. In each case, the first two search levels use full-breadth h -refinements; all remaining levels (3-7) use variable-breadth, hybrid h - p refinements, which are pruned to an increasingly tight focus, with increasing search ply depth. A minimum energy h - p AFEA trajectory tree pruning criterion was used for each scheme. Increasing the search depth beyond 7-ply, yields increasingly reduced efficacy, in elapsed runtime. Note: Mesh n is a discretization that contains n finite elements.

TABLE I
PARALLEL PROCESSES AND RUNTIME OF TUNED-BREADTH AFEA

Search Ply Depth per Step	1	2	3	5	7
Mesh n Parallel Processes	n	$n(n+3)/2$	$\sim 5n^2$	$\sim 42n^2$	$\sim 480n^2$
Normalized Elapsed Time	1	~ 0.5	~ 0.3	~ 0.2	~ 0.14

REFERENCES

- [1] G. Chen, H. Zhu, T. Cui, Z. Chen, X. Zeng and W. Cai, “ParAFEMCap: a parallel adaptive finite-element method for 3-D VLSI interconnect capacitance extraction”, *IEEE Transactions on Microwave Theory and Techniques*, Vol. 60, No. 2, pp. 218-231, Feb. 2012.
- [2] K. Key and J. Oval, “A parallel goal-oriented adaptive finite element method for 2.5-D electromagnetic modelling”, *Geophysical Journal International*, Vol. 186, pp. 137-154, July 2011.
- [3] T. Cui, “Efficient parallel adaptive computation of 3D time-harmonic Maxwell’s equations using the toolbox PHG”, *IEEE Int. Conf. on High Performance Computing and Communications*, pp. 876-881, 2008.
- [4] S. McFee and D. Giannacopoulos, “The implications of parallel processing on h - p adaptive finite element analysis for electromagnetics”, *IEEE Transactions on Magnetics*, Vol. 34, No. 5, pp. 3284-3287, Sept. 1998.