A Numerical Computation Model of Electrical Impedance Tomography Forward Problem Based on Generalized Finite Element Method

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***Abstract*—In Electrical Impedance Tomography (EIT), one of the major problems of complex geometry shape is its high demand in computation capability power and memory. To calculate the forward problem accurately, a Generalized Finite Element Method (GFEM) is proposed to overcome the limitation. Then a smaller number of nodes and elements compared with conventional FEM are needed with a numerical computation model of EIT. In the forward solution, it is capable of achieving better accuracy with less computational time and memory. Our results demonstrate the efficiency of the GFEM in EIT simulation.**

***Index Terms*—Bioimpedance, Biomedical computing, Biomedical engineering, Biomedical image processing, Computational electromagnetics**

I. Introduction

Electrical impedance tomography uses electrodes placed on the surface to make measurements and then an image of the electrical conductivity distribution within the body is reconstructed with an algorithm. It is a relatively novel low-cost non-invasive imaging technique that has evolved over the past 30 years [1]. And EIT shows the potential to be of great value in clinical diagnosis [2-3].

In electrical impedance tomography (EIT), a numerical computation forward problem model with capable of predicting the voltages on surface electrodes for a given conductivity distribution is indispensable for image reconstruction.

The EIT forward problem model is normally based on the conventional Finite Element Method (FEM) [4-6]. One of the major problems of complex geometry shape or 3-D EIT is its high demand in computation capability power and memory. High precision both in numerical computation and in data acquisition is required for obtaining the reconstruction images for a small anomaly in the computing domain. In our work we address the problem of calculating the forward problem accurately. The Generalized Finite Element Method (GFEM) [7-8] is proposed to overcome this limitation. With the introduction of GFEM, a smaller number of nodes and elements compared with conventional FEM are needed. In the forward solution with GFEM, it is capable of achieving better accuracy with less computational time and memory. The results demonstrate the efficiency of the GFEM in EIT simulation.

II. Formulation

A. Generalized Finite Element Method

The Generalized Finite Element Method comes from manifold method [7] is a developed general method to analyze material response to external and internal changes in stress originally. And now it has been used in electromagnetic fields computation and analysis [8]. In this method, the node is generalized, and so it can have more than two or three generalized degrees of freedom, and those degrees of freedom are not required to have their own definite physical meaning necessarily. At each generalized node, we can take a polynomial to define a generalized type of nodal interpolation function.

Let us suppose S^h is the conventional FEM space, and a Lagrange interpolation function $\left[\varphi_1\ \varphi_2\ \ldots\ \varphi_\mathrm{n}\right]^\mathrm{T}$ is used, then the field variable U^h can be written as a summation with the conventional as the FEM: $\mathrm{U}^h=\sum_{i=1}^{N}\vec{u}_i\vec{\varphi}_i$, where the $\vec{u}_i$ (i= 1,2,⋯,N) is the vector of degrees of freedom $\left[u_i\ v_i\right]^T$ on the i^{th} node which represents the potential variation on the node. When the node is generalized it can have more degrees of freedom, and those degrees of freedom

$$\vec{u}_i=\begin{Bmatrix}u_i\\ v_i\end{Bmatrix}=\sum_{j=1}^{m_i}\begin{bmatrix}f_{ij}(x,y) & 0\\ 0 & f_{ij}(x,y)\end{bmatrix}\begin{Bmatrix}d_{i,2j-1}\\ d_{i,2j}\end{Bmatrix}.$$

Then we get the following equation from the above,

$$\begin{aligned}\mathrm{U}^h&=\sum_{i=1}^{N}\sum_{j=1}^{m_i}\begin{bmatrix}f_{ij}(x,y) & 0\\ 0 & f_{ij}(x,y)\end{bmatrix}\begin{Bmatrix}d_{i,2j-1}\\ d_{i,2j}\end{Bmatrix}\vec{\varphi}_i\\ &=\sum_{i=1}^{N}\vec{\varphi}^i\vec{F}^i\vec{D}^i=\sum_{i=1}^{N}\vec{N}^i\vec{D}^i\end{aligned},$$

where $\vec{N}^{\mathbf{i}}$ is the matrix of interpolation function which has its origin $\vec{\varphi}_i$, and $\vec{D}^i$ is the generalized vector of degrees of freedom with the form of $\vec{\mathbf{D}}^i=\left[d_{i,1}\ \ d_{i,2}\ \ \ldots\ \ d_{i,2mi}\right]^T$.

When a zero-order generalized nodal interpolation function is used, the GFEM would be reduced to conventional FEM.

B. EIT Forward Problem

A low-frequency EIT forward problem is modeled as (1). The electric field is conservative, and the conduction currents dominant with respect to their displacement counterparts lead to the equation:

$$\nabla \cdot \rho^{-1} \nabla \phi = 0 \qquad \text{in}\, \mathrm{S}^{\mathrm{h}} \tag{1}$$

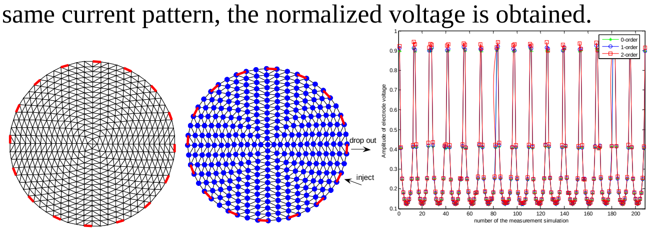
where ∇ is the gradient operator, $\nabla\phi$ represents the static electric field; ρ is the resistivity of the body; ϕ is the electric potential; S^{h} represents the body to be imaged. Electrodes are modeled with boundary conditions as the complete electrode model [9]. For one triangular element, there are three generalized nodes. Then the field variable $\mathbf{U}^h$ in the element could be written as $\mathrm{U}^h{}_e = \sum_{i=1}^{N} \vec{N}^i{}_e \vec{D}^i{}_e$, and we get $\vec{N}^i{}_e = \vec{\varphi}^i{}_e F^i \qquad (i = 1,2,3)$.

For EIT problem, it is not easy to derive the governing equations of the GFEM with variational principles. So the method of weighted residuals is implemented to derive the governing equations.

III. Simulation Results

To validate the results of GFEM, a circle forward model with 16 electrodes is used. The circle has radius of 1 m. The 16 electrodes attached on its boundary as shown in Fig. 1. In Fig. 1. (a) is the zero-order GFEM model and it is a conventional FEM model which contains 545 nodes, 1024 elements; and (b) is the one-order and two-order GFEM model which contains 313 generalized nodes, 576 elements.

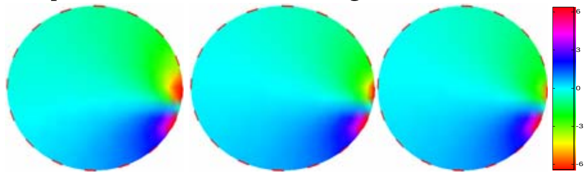
The contact impedances of the electrodes is set to 0.01 $\Omega \cdot \mathrm{m}^2$ in the simulation. The adjacent pair current patterns and adjacent measurements protocols are used. In Fig. 1 (c), the normalized voltage values of electrodes results are shown. Dividing the maximum value of voltage on electrode in the same current pattern, the normalized voltage is obtained.



(a) (b) (c)

Fig. 1. Two GFEM models
(a). zero-order GFEM, (b) one/two-order GFEM model, (c)Normalized voltage values of electrodes measurement

The simulation result is obtained using the zero, one and two-order GFEM basis functions. The results show that the three orders agree very well with the L-2 norm error less than 0.0026. The computed voltage with different orders GFEM for one pattern is demonstrated in Fig. 2.



(a) (b) (c)

Fig. 2. Computed voltage with different orders GFEM for one current pattern (a)zero-order, (b) one-order, (c) two order

We could obtain the same even better computed results with less nodes when one-order or two-order GFEM is used.

IV. Conclusion

The generalized finite element method has been developed and validated for the EIT forward model. Numerical simulation results show that GFEM is able to achieve the same or better accuracy with conventional FEM. In summary, we have shown the GFEM is an efficient and promising method in forward problem solution for electrical impedance tomography.

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