Approximate and proper electromagnetic modeling in moving conductors

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***Abstract*—A conductor moving in a stationary magnetic field often rises cruical issues in the courses on electromagnetics for electrical engineering students. The correct use of Faraday's induction law can sometimes be harder than one would think for the first sight. In this paper, we revisit a well-known demonstration example of eddy-currents by means of numerical field computation: the case of a small magnet falling within a copper tube is dealt with by approximate and proper electromagnetic models. The approximate solutions are usually of satisfying accuracy, but they hide some parts of the physics behind the phenomenon. At the university courses, however, the deep understanding of the electromagnetics must proceed the use of practical simplifications, even when using up-to-date numerical field computation softwares.**

***Index Terms*—education, Faraday's law, moving conductor, finite element method**

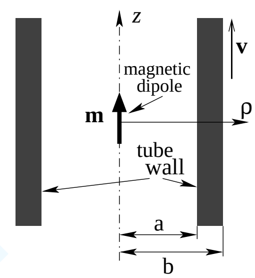

Figure 1: The axisymmetric configuration in a cylindrical coordintate system (z, ρ, φ). A magnetic dipole (with a z-directed moment $\mathbf{m}$) is at rest on the axis of an infinite-long tube which moves with a velocity $\mathbf{v}$.

I. Introduction

One of the most impressive demonstrations of the eddy-currents is the damped fall of a strong magnet in a non-ferromagnetic conducting tube. The magnet's terminal velocity is much smaller than in free-space due to the braking effect of the induced eddy-currents in the tube wall. This experiment is perfect for focusing the young students' interest on electromagnetic phenomena and also for teaching quantitaive modeling for graduate students.

Several analytical (e.g., [1], [2], [3]) and experimental (e.g., [4]) approaches have recently been published on this demonstration example. A common concern about these works is that they consider the magnetic field generated by the falling magnet only and neglect the magnetic field risen due to the currents induced in the tube wall. This second part of the induction is much smaller than the first one in the standard configurations at relatively "small" falling velocities. However, in the viewpoint of the education, the proper modeling (even if it is complicated), might sometimes be more useful than a good approximation (which can easily be misunderstood by the students). A common mistake in students' thinking is to force sequential rules even if there is no distinguished order of the phenomena but they all interact with each other. A such example with a "reaction-effect" might help the students to see the electromagnetics clearer.

The assumption of "small" speed –as a condition for the neglection of the self-inductance phenomenon– also occurs in other common examples (not considered herein), like the infinite metal plate moving below a strong magnet ([5], [6]). Within the frame of the Lorentz Force Velocimetry ([7]), again similar approximations are usually made.

In this paper, we present the EM modeling of the magnet falling in a conductive tube and study the relation between the results obtained by the approximate and the proper model, with respect to the velocity of the fall. The proper model requires numerical field computation; we use the Finite Element Method.

II. The studied configuration and the EM models

The copper tube (conductivity: $\sigma = 57\,\mathrm{MS/m}$) is vertical, the inner and outer radii are $a = 7.85\,\mathrm{mm}$ and $b = 9.75\,\mathrm{mm}$, respectively. The tube is very long and the steady state is studied: the magnet falls with its terminal velocity v (the sum of all forces acting on the magnet –gravity, drag and magnetic– gives zero) and v is assumed to be known. The magnet is assumed to be small, i.e., it is modeled by a magnetic dipole with a vertical moment, moving on the axis of the tube (z), see Fig. 1.

In the model, the magnet is fixed to the center of the cylindrical coordinate system and the tube is assumed to move to the $+z$ direction with a velocity v. Let us denote the magnetic induction of the dipole by $\mathbf{B}_0$ (expression is avaliable in textbooks).

Our goal is to obtain the current density within the tube wall. The constitutive relation in the moving conductor[1]:

$$\mathbf{J} = \sigma\mathbf{E}' = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \tag{1}$$

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[1] For the sake of rigour: v has to be much smaller than the speed of light.

where $\mathbf{E}'$ is the electric field in the moving media and $\mathbf{E}$ is the electric field in the rest-frame. The latter is zero, since no static charge is experienced anywhere in the conductor, due to the axial symmetry of the configuration. $\mathbf{J}$ has an azimuthal component only. Equation (1) includes $\mathbf{B}$, which is the total magnetic induction in the rest-frame.

Let us assume $\mathbf{B} = \mathbf{B}_0$, i.e., neglect the induction associated with the current in the tube wall. In so doing, the current density –now denoted by $\mathbf{J}_0$– can easily be expressed:

$$\mathbf{J}_0 = \sigma \mathbf{v} \times \mathbf{B}_0 = \hat{\mathbf{e}}_\varphi \sigma v B_{0,\rho}, \tag{2}$$

where $B_{0,\rho}$ is the radial component of the induction of the dipole.

The total induction is, however, $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_e$. Let us derive the so far neglected second term from a vector potential: $\mathbf{B}_e = \nabla \times \mathbf{A}$. This potential satisfies the Laplace-Poisson equation (with the gauge $\nabla \cdot \mathbf{A} = 0$):

$$\Delta \mathbf{A} = -\mu_0 \mathbf{J}. \tag{3}$$

Rewriting this into (1), we get:

$$-\Delta \mathbf{A} - \mu_0 \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) = \mu_0 \sigma (\mathbf{v} \times \mathbf{B}_0). \tag{4}$$

Since $\mathbf{B}_e$ has axial (z) and radial (ρ) components only, $\mathbf{A}$ is azimuthal: $\mathbf{A} = A(z, \rho)\hat{\mathbf{e}}_\varphi$. The differential equation for A is:

$$-\frac{\partial}{\partial \rho}\left(\rho \frac{\partial A}{\partial \rho}\right) - \frac{\partial}{\partial z}\left(\rho \frac{\partial A}{\partial z}\right) + \frac{A}{\rho} + \rho \mu_0 \sigma v \frac{\partial A}{\partial z} = \rho \mu_0 \sigma v B_{0,\rho}. \tag{5}$$

In the air-filled regions inside and outside the tube, $\sigma = 0$ is set in (5). A is continuous at the boundaries and vanishes at infinity.

Equation (5) is solved by the Finite Element Method. In the PDE-toolbox of Matlab®, the elliptic equation scheme can be used, but the term containing $\frac{\partial A}{\partial z}$ is put to the right side and the equation is solved as a nonlinear one.

Once A is obtained, the current density is given by (3).

III. Results

The preliminary numerical studies justify the expectations: the discrepancy between the results of the approximate and the proper model gets larger as the velocity increases. For the configuration described in the previous section, the current densities are plotted for two velocities in Fig. 2. Considering that the typical velocities in such experiments are smaller than 2 m/s, even the approximate model provides satisfying results. However, one has to know the limitations of the approximation. At $v = 10\,\mathrm{m/s}$, a significant difference is experienced between the results. The current distribution is not symmetric to the origin, in contrast with the prediction of the approximate model.

IV. Conclusion

A numerical simulation of the eddy-current distribution due to a moving magnet within a conducting tube wall is presented. The proper model takes into account not only the magnetic field of the magnet but the field risen by the induced current as well. So far a known velocity has been assumed. As v occurs in the coefficients of the partial differential equation, this method cannot be used for the determination of v directly. In the full version of the paper, we present an iterative scheme for the computation of v, when the magnetic moment and the weight of the small magnet is known.

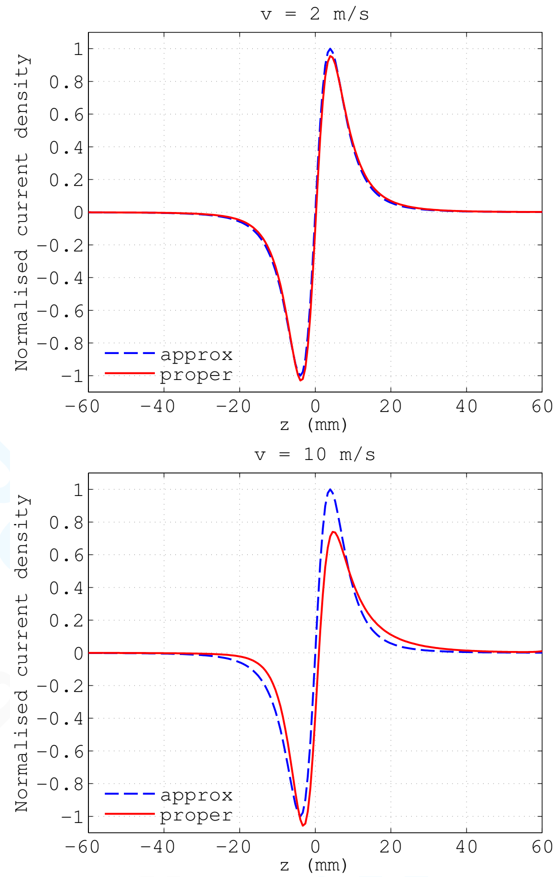

Figure 2: Normalised current densities on the inner wall of the tube at two different velocities.

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