

# Error estimation in the Computation of Induced Current of Human Body in the Case of Low frequency Magnetic Field Excitation

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**Abstract** — Computation of induced current in human body with FEM requires an important computation time. To avoid this difficulty the  $\phi$ -A formulation has been proposed and its efficiency have been demonstrated. Nevertheless as the accuracy of the solution is in direct link with the mesh we propose to evaluate the numerical solution with a *posteriori* error estimator.

## I. INTRODUCTION

In order to compute the current density induced in the human body in very low frequencies (i.e. less than 1 MHz) some methods are proposed in the literature [1-3]. The major issues with this modeling are the determination of the dielectric properties of the tissues [7], the estimation of the quality of the solution and, when using the Finite Element method, the creation of the mesh for the computational phantom. Another problem in link with the mesh and the quality of the solution is the time computation: so as to reduce the computation time a dedicated  $\phi$ -A formulation which takes into account the specificities of the human body has been proposed [1]. In order to limit the mesh density, and consequently the number of unknowns and the computation time, error estimator can be used. Many error estimators are proposed today [4-6]: some of them are based on a *posteriori* techniques. With these approaches the estimated error map could be used for an adaptive mesh refinement. One of them called *residual based error estimator* provides an estimation of the local numerical error: in this work we propose to analyze the quality of the mesh in the case of the dedicated  $\phi$ -A formulation by using this error estimator. As example, we compute the induced current density in the human body for a magnetic excitation by using two heterogeneous (i.e. including several tissues with their own different conductivities) meshes of the human body of respectively  $7 \cdot 10^5$  and  $10^7$  elements.

## II. NUMERICAL METHODS

### A. $\phi$ -A formulation

We consider a human body submitted to a known magnetic flux density source  $\mathbf{B}_s$  in low frequency, and let  $\mathbf{A}_s$  such that  $\mathbf{B}_s = \text{curl} \mathbf{A}_s$  is a source magnetic vector potential. Due to the low frequency and to the dielectric properties of the tissues which compose the human body, it can be assumed that: i) displacement currents are negligible, ii) the distribution of the magnetic field is not affected by the induced currents. Basing on these assumptions, the  $\phi$ -A formulation [3] has been developed:

$$\text{div} \mathbf{J} = \text{div} [-\sigma(\mathbf{j}\omega \mathbf{A}_s + \text{grad} \phi)] = 0 \quad ; \quad \mathbf{n} \cdot \mathbf{J}|_{\mathcal{D}} = 0 \quad (1)$$

which can be written in weak form as:

find  $\phi \in H(\text{grad}, \mathcal{D})$  such that:

$$(\sigma(\mathbf{j}\omega \mathbf{A}_s + \text{grad} \phi), \text{grad} \phi')_{\mathcal{D}} = 0 \quad \forall \phi' \in H(\text{grad}, \mathcal{D})$$

where the symbol  $(\cdot, \cdot)$  means integration over the domain  $\mathcal{D}$ ,  $\phi$  is an unknown electric scalar potential,  $\mathcal{D}$  is the computational domain (which is bounded to the human body) and the space  $H(\text{grad}, \mathcal{D})$  is defined as:

$$H(\text{grad}, \mathcal{D}) = \{\phi \in L^2(\mathcal{D}) : \|\text{grad} \phi\|_{L^2}^2 < \infty\}$$

The discrete form of this formulation writes:

find  $\phi_h \in H_h(\text{grad}, \mathcal{D})$  such that:

$$(\sigma(\mathbf{j}\omega \mathbf{A}_{sh} + \text{grad} \phi_h), \text{grad} \phi'_h)_{\mathcal{D}} = 0 \quad \forall \phi'_h \in H_h(\text{grad}, \mathcal{D})$$

where  $\phi_h$  is expanded with nodal functions. We recall that  $\mathbf{A}_s$  and consequently its discrete form is a vector potential source term which is a data of the problem, which can be obtained previously by a Finite Element computation or by an analytical solution.

### B. Residual based error estimator

Let  $\phi$  be the exact solution of the problem and  $\phi_h$  the numerical one using the finite element method. So, the error  $e_\phi$  on  $\phi$  takes the form:

$$e_\phi = \phi - \phi_h \quad (2)$$

Now the global energy norm error of our numerical discretization (denoted by  $\varepsilon_\omega^2$ ) is given by:

$$\varepsilon_\omega^2 = \int_{\omega} |(\mathbf{j}\omega \mathbf{A}_s + \text{grad} \phi) - (\mathbf{j}\omega \mathbf{A}_{sh} + \text{grad} \phi_h)|^2$$

The local error estimator on the element  $T$  is given by [5]:

$$\eta_T^2 = \eta_{T;1}^2 + \sum_{F \in \mathcal{T} \cap \Gamma_{\text{in}}} \eta_{F;1}^2 + \sum_{F \in \mathcal{T} \cap \partial \Omega} \eta_{F;2}^2$$

with:

$$\eta_{T;1} = h_T \|\text{div}(\sigma(\mathbf{j}\omega \mathbf{A}_{sh} + \text{grad} \phi_h))\|_T,$$

$$\eta_{F;1} = h_F^{1/2} \left\| \left[ \sigma(\mathbf{j}\omega \mathbf{A}_{sh} + \text{grad} \phi_h) \cdot \mathbf{n} \right]_F \right\|_F,$$

$$\eta_{F;2} = h_F^{1/2} \left\| \sigma(\mathbf{j}\omega \mathbf{A}_{sh} + \text{grad} \phi_h) \cdot \mathbf{n} \right\|_F,$$

where  $h_T$  and  $h_F$  are denoted respectively by the diameter of the element  $T$  and face  $F$ . It can be shown that the local equation is evaluated by  $\eta_{T;1}$ ,  $\eta_{F;1}$  verifies the discontinuities of the eddy current, and  $\eta_{F;2}$  evaluates the

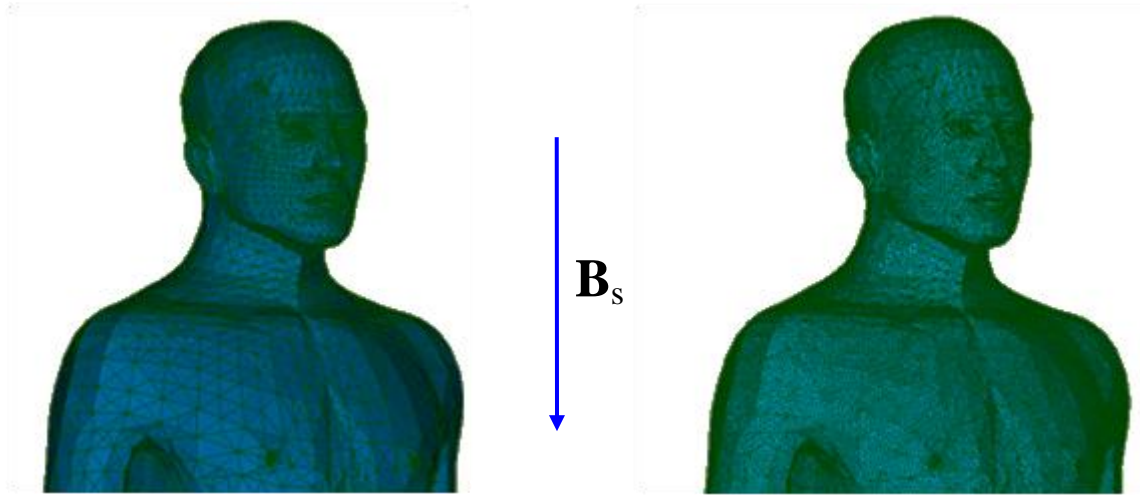


Figure 1: Coarse mesh (a) and fine mesh (b)

boundary condition on  $\partial\mathcal{D}$ . This estimator provides a local estimate of the error, up to an unknown multiplicative constant: thus, the estimator maps provide only *relative* information about the error in different locations of  $\mathcal{D}$ .

### III. APPLICATION

We computed the induced current  $\mathbf{J}$  by using two meshes of human body, respectively of  $7 \cdot 10^5$  and  $10^7$  elements, with a 50 Hz uniform source magnetic flux density of  $50\mu\text{T}$  vertically oriented (figure 1). The values of the global error estimator are  $5.07 \cdot 10^{-5}$  for the coarse mesh and  $1.255 \cdot 10^{-5}$  for the fine mesh. Clearly, the estimate is lower for the fine mesh: in fact, the ratio of global error estimates corresponds to the average ratio of the element sizes between the two meshes (roughly 5:1). Then we have determined the error map by using the proposed error estimator, which is depicted in figure 2 and 3 for the brain and the eyes respectively. It can be observed that for both these organs the distribution of the error map is more uniform in the case of the fine mesh.

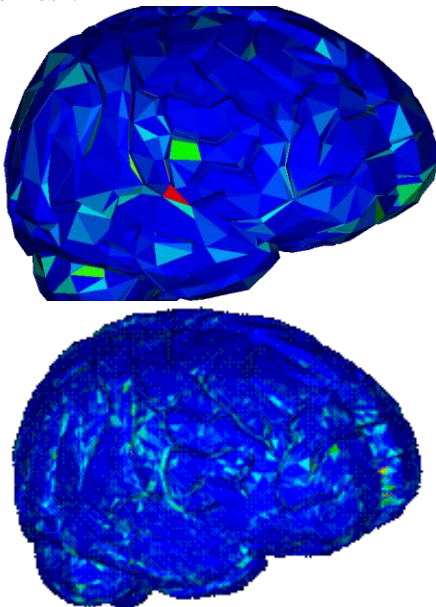


Figure 2: Error obtained using the error estimator for coarse mesh (top), and fine mesh (bottom)

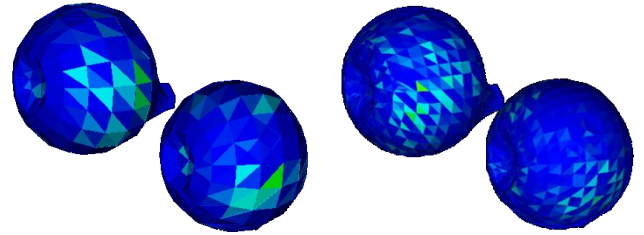


Figure 3: Error obtained using the error estimator for coarse mesh (left), and fine mesh (right)

### IV. CONCLUSION

In this paper to estimate the quality of the solution in the case of the induced currents in the human body we have proposed an error estimator. This estimator has been used in the case of  $\phi$ - $\mathbf{A}$  formulation. The use of this estimator to analyze the results obtained from two meshes of the human body makes obvious these possibilities.

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