Fast Multipole Method accelerated meshfree Post-Processing in 3D Boundary Element Methods

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***Abstract*—A bidirectional coupling of a boundary element method with a visualization tool is a very efficient and flexible post-processing approach. Field values are only computed in points of visualization objects, e. g. points of field lines, and the amount of stored data along with computational costs are kept as small as possible. Furthermore, a meshfree post-processing suggests itself in the case of boundary element methods, since field values in arbitrary points are directly obtained from boundary integrals. Established compression techniques like the fast multipole method reduce significantly the computational costs of boundary integral computations. Here, a modified fast multipole method based approach is presented for an efficient meshfree post-processing of three-dimensional field problems. A flexible octree scheme is applied to treat evaluation points, which are determined during the computation of visualization objects. Furthermore, the number of evaluations of multipole expansions is reduced by the presented reversed fast multipole algorithm.**

***Index Terms*—boundary element methods, fast multipole method, meshfree methods, visualization methods**

I. Introduction

Visualization is an established technique for the post-processing of three-dimensional electromagnetic field problems. Especially, virtual reality and augmented reality enable a deep insight into the studied problem along with a visualization, which is easy to understand [1, 2]. A bidirectional coupling of a field computation tool with a visualization tool is a sound basis for an interactive and powerful post-processing [3]. Field values are only computed in points, which are necessary for the displayed visualization objects. Then, expensive pre-computations of field values in the complete considered domain are avoidable and the amount of stored and transferred data is dramatically reduced. Especially in the case of boundary element methods (BEM), a bidirectional coupling of meshfree post-processing with a visualization tool is very advantageous. For instance, the computation of field lines results in the evaluation of boundary integrals in a very small number of evaluation points along with a high accuracy [4]. Moreover, an efficient and accurate application of modern visualization techniques like topology detection [5] is enabled, too.

An application of the fast multipole method (FMM) to a BEM compresses the system of linear equations and reduces the computational costs of post-processing dramatically [6]. However, the approach presented in [6] requires that the evaluation points are known in advance. Here, a modified FMM algorithm is presented, which enables the determination of evaluation points during the post-processing and efficient field computations in single points. A flexible octree, which is similar to the one of automatic domain detection methods [7], is used. The number of computations of multipole expansions is reduced by a reversed FMM algorithm, which is described in detail in the following section. A combination of an efficient automatic domain detection and the shown modified FMM algorithm result in a very flexible and powerful post-processing in BEM.

II. Formulation

The algorithmic basis of the FMM is an octree scheme. All N_{be} boundary elements are spatially grouped using hierarchical octree cubes. An evaluation point $\boldsymbol{r}_{ep}$, which is defined by its Cartesian coordinates, is added during runtime of the post-processing to the flexible octree scheme, which is described in detail in the first sub-section. Field values in the evaluation point are computed based on a modified reversed FMM algorithm, which is discussed in the second sub-section.

A. Flexible octree scheme

A demand on the flexible octree is that it supports dynamic changes of its cube structure due to added evaluation points. Furthermore, its cube structure must be optimized to the concrete task. Hence, a separate octree for the solution of the linear system of equations, for automatic domain detection, and for the presented post-processing computations is recommended. Then, important parameters like the maximum number of boundary elements in a cube or the maximum real size of bounding boxes of cubes can be adjusted problem oriented. Of course, the code basis is the same in all cases but adapted to the specific task using modern software techniques.

Operations based on the octree scheme are optimized, if only relevant objects are added to the octree. That means, a separate octree is created for each computational domain of the applied BEM formulation and only boundary elements, which belong to the computational domain, are considered. Depending on the used BEM formulation, the computational domain is equivalent to the spatial domain, to which the evaluation point belongs, or the computational domain corresponds to the total space. For instance, all boundary elements are taken into account in the case of an indirect electrostatic BEM formulation but in the case of direct electrostatic BEM formulation only boundary elements of the surface of a single domain are chosen. An automatic domain detection is required to obtain both the computational domain of the evaluation point and material data in the evaluation point.

After the initialization of the octree with all relevant boundary elements, the evaluation point is added similar to the domain detection approach described in [7]. If the evaluation point lies outside the current root cube of the octree, the octree is enlarged by adding a new root cube. Then, the evaluation point is assigned to a cube C_{ep} taking into account the following rules.

The most expensive part of the FMM is the computation of so-called near-field interactions, which are classical boundary integrals taking into account all boundary elements of the cube C_{ep} and of all direct neighbors of C_{ep}. Therefore, the number of boundary elements in the near-field of the evaluation point has to be minimized by properly chosen octree parameters and optimized octree creation. Furthermore, the local expansion of C_{ep}, which represents all far-field interactions, should be reused for a large number of evaluation points to reduce the number of computations of series expansions. Hence, the largest possible cube C_{ep} is searched that has no boundary elements and that has no direct neighbors with boundary elements. Note boundary elements that stick into a cube must be considered, too. Furthermore, the cube C_{ep} is only sub-divided into smaller child cubes, if C_{ep} is not smaller than the smallest neighbor of C_{ep}, since for an application of series expansions of the FMM the convergence radius of the larger cube is relevant.

B. *Modified FMM algorithm*

The classical FMM algorithm starts at the finest octree level and computes multipole coefficients of octree cubes directly from sources on the boundary elements, for instance converts surface charge densities into multipoles. Then, multipole coefficients of child cubes are transformed into multipole coefficients of their parent cube. The coefficients of the local expansion, which is a Taylor series expansion in spherical coordinates, are obtained from multipole coefficients of cubes in the far-field. These transformations are very expensive and their number should be limited. Local coefficients of a large cube are converted into local coefficients of child cubes. Finally, the local expansion is evaluated in the evaluation points and the near-field interactions are added to the result.

The above-described algorithm works very well, if all evaluation points are known in advance, for instance during the solution of the linear system of equations or in the case of field computations in the nodes of a given post-processing mesh. Here, a modified reversed FMM algorithm is preferred. It starts at the evaluation point. If boundary elements are lying in the cube C_{ep} or in its direct neighbors, near-field interactions are computed first. There, singular or nearly singular integrals are evaluated using a general implementation to take into account all kind of boundary elements. Then, the local expansion of the cube C_{ep} is evaluated to consider far-field interactions. The coefficients of the local expansion are computed from the local coefficients of the parent cube C_{pep} and from the multipole coefficients of the children of the direct neighbors of C_{pep}, which are not direct neighbors of C_{ep}. The multipole coefficients of a cube are computed from the multipole coefficients of its child cubes or directly from the sources on the boundary elements of the cube, respectively.

The main advantage of the presented modified reversed FMM algorithm is that multipole coefficients and local coefficients are computed only on demand. Only necessary series expansions are computed and not series expansions of all cubes. Hence, the number of expensive transformations of series expansions is as small as possible.

In practice, the next evaluation point lies often close to a previous evaluation point. Then, already computed coefficients of series expansions can be reused and only near-field interactions must be computed again.

III. Numerical Example

The electric field of the antennas of an electronic musical instrument called Theremin is studied using an indirect electrostatic BEM formulation. The pitch antenna is used to adjust the frequency and the volume antenna is used to adjust the volume of a tone. Both antennas are part of an oscillatory circuit and are considered as capacitors. Hence, an electrostatic field computation suffices.

The surfaces of casing of the Theremin, which is made of wood, the surfaces of the antennas and of the grounded electrode inside the casing, and the surface of the hand of the musician are modeled using 1795 second order, quadrilateral elements (Fig. 1a)). The system of linear equations with 5553 unknowns has been solved on an Intel Core i7-3820QM within 30 s and the potential in 15402 evaluation points in a cutting plane has been computed in 3 s (Fig. 1b)). The computation of field lines including a visualization with augmented reality will be shown in the full paper.

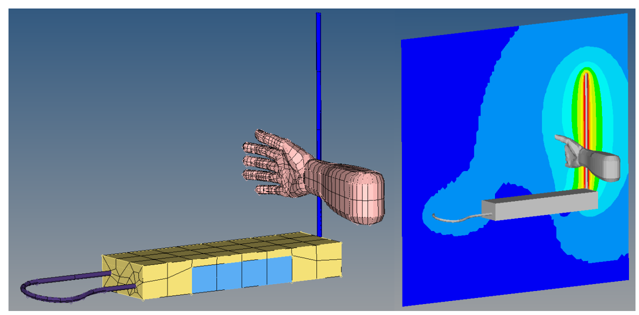

Fig. 1: a) Boundary element mesh of a Theremin including hand of musician, b) Electric potential of the antennas of a Theremin

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