A Split Step Precise Integration Time Domain Method and its Numerical Dispersion

Qi Liu, Xikui Ma, Xu Zhuansun
The State Key Lab of Electrical Insulation and Power Equipment, School of Electrical Engineering,
Xi'an Jiaotong University, Xi'an, Shaanxi 710049, CHINA
qliucn@live.cn

***Abstract*—A modified precise integration time domain (PITD) method is proposed for solving Maxwell's equations. Due to the use of the split step scheme, this method can reduce the requirement of computation time and storage space in comparison with the conventional PITD method. The numerical dispersion relation is derived and analyzed. It is shown that this method has better dispersion performances in comparison with the alternating direction implicit finite difference time domain (ADI-FDTD) method. Numerical examples are given to verify the accuracy and memory efficiency of this method.**

***Index Terms*—Finite difference methods, Maxwell equations, time domain analysis.**

I. Introduction

In order to improve the computation efficiency of the finite difference time domain (FDTD) method, the precise integration time domain (PITD) method has been proposed [1]. In this method, the central difference scheme is used to discretize Maxwell's equations in space, and the precise integration technique is used to advance the field components in time. Although the Courant-Friedrich-Levy (CFL) condition is not totally removed, the PITD method can maintain stability despite using a time step size much larger than the CFL limit [2]. Besides, compared with the unconditional stable methods, such as alternating direction implicit (ADI) [3], [4], locally 1-D (LOD) [5], [6] and split step (SS) [7], the PITD method has lower numerical dispersion errors, and its numerical dispersion errors can be made nearly independent of the time step size. These properties make the PITD method especially suitable for the problems where very fine meshes with respect to the wavelength are required. However, the PITD method has the drawback of high memory requirements when analyzing large geometries, which may limit its further applications.

In this paper, based on the split step scheme [7], a modified version of precise integration time domain method, referred as SS-PITD, is proposed. In it, the conventional PITD calculation is factorized into two sub-steps procedures, and each one only needs the solution of a number of 1-D wave equations. This may lead to a significant reduction in the computation time and memory usage.

II. Formulations

For simplicity, the formulations of SS-PITD method are presented for a two dimensional (2-D) case. However, the 3-D problems can be handled in the same manner. The Maxwell's curl equations for the TE wave in an isotropic, lossless medium can be written in a compact matrix form as

$$\frac{\partial}{\partial t}\mathbf{U} = (\mathbf{A}+\mathbf{B})\mathbf{U} \tag{1}$$

where $\mathbf{U} = [\mathbf{E_x}, \mathbf{E_y}, \mathbf{H_z}]^T$, and

$$\mathbf{A} = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \dfrac{\partial}{\varepsilon\partial x} \\ 0 & \dfrac{\partial}{\mu\partial x} & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & \dfrac{\partial}{\varepsilon\partial y} \\ 0 & 0 & 0 \\ \dfrac{\partial}{\mu\partial y} & 0 & 0 \end{bmatrix} \tag{2}$$

In the conventional PITD method [1], the recursive solution of (1) marching from the n th time step to the $(n+1)$ th time step can be written as

$$\mathbf{U}^{n+1} = e^{(\mathbf{A}+\mathbf{B})\Delta t}\mathbf{U}^n \tag{3}$$

where Δt is the time step size, n is the time index. The matrix exponential of (3) can be approximated by

$$e^{(\mathbf{A}+\mathbf{B})\Delta t} \approx e^{\mathbf{A}\Delta t}e^{\mathbf{B}\Delta t} \tag{4}$$

Thus, by using the split step scheme, the PITD time step Δt is divided into two sub-steps with the time increments of $\Delta t/2$, and the numerical solution of (1) can be obtained by successively solving

$$\frac{\partial}{\partial t}\mathbf{U} = 2\mathbf{AU} \quad \text{and} \quad \frac{\partial}{\partial t}\mathbf{U} = 2\mathbf{BU} \tag{5}$$

Similar to the conventional PITD method, the computation region is discretized by the Yee's grid, and the spatial derivatives is approximated by the central difference scheme. Then, (5) is reduced to two systems of ordinary differential equations (ODEs)

$$\frac{d}{dt}E_x\Big|_{i+(1/2),j} = 0 \tag{6a}$$

$$\frac{d}{dt}E_y\Big|_{i,j+(1/2)} = -\frac{2}{\varepsilon\Delta x}\left(H_z\Big|_{i+(1/2),j+(1/2)} - H_z\Big|_{i-(1/2),j+(1/2)}\right) \tag{6b}$$

$$\frac{d}{dt}H_z\Big|_{i+(1/2),j+(1/2)} = -\frac{2}{\mu\Delta x}\left(E_y\Big|_{i+1,j+(1/2)} - E_y\Big|_{i,j+(1/2)}\right) \tag{6c}$$

and

$$\frac{d}{dt}E_x\Big|_{i+(1/2),j} = \frac{2}{\varepsilon\Delta y}\left(H_z\Big|_{i+(1/2),j+(1/2)} - H_z\Big|_{i+(1/2),j-(1/2)}\right) \tag{7a}$$

$$\frac{d}{dt}E_y\Big|_{i,j+(1/2)} = 0 \tag{7b}$$

$$\frac{d}{dt}H_z\Big|_{i+(1/2),j+(1/2)} = \frac{2}{\mu\Delta y}\left(E_x\Big|_{i+(1/2),j+1} - E_x\Big|_{i+(1/2),j}\right) \tag{7c}$$

where Δx and Δy are the space step sizes along x- and y-directions, respectively. It can be seen that only two field components relating to the spatial difference along one direction are needed to be updated in each sub-step. Thus, the

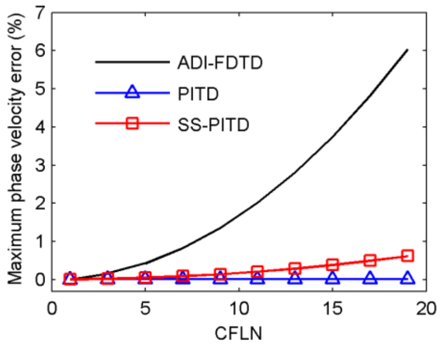

Fig.1 Maximum phase velocity error versus CFLN with λ / Δ =100

TABLE I
COMPUTATIONAL RESULTS

Method	CFLN	Result(GHz)	Error	Memory(MB)
FDTD	0.5	16.16	0.40%	0.03
PITD	2	16.14	0.56%	107.9
ADI-FDTD	2	15.73	3.04%	0.05
SS-PITD	2	15.99	1.44%	0.17

original 2-D problem is reduced to a number of 1-D problems, and then they are solved by the precise integration technique.

III. Numerical Results

To demonstrate the superiority of the SS-PITD method, the numerical dispersion characteristics of the proposed method are investigated. For clarify, a uniform mesh ($\Delta x = \Delta y = \Delta$) are employed, we define CFLN as the ratio of the time step to the CFL limit. Fig. 1 shows the maximum numerical phase velocity errors versus CFLN for the ADI-FDTD, the conventional PITD and the SS-PITD method with λ / Δ =100. Due to the error caused by splitting the matrix exponential, the maximum phase velocity errors of the SS-PITD method increase as CFLN increases, but they can be much smaller than those of the ADI-FDTD method.

Fig.2 shows a common model in the electromagnetic compatibility (EMC) issues. The upper portion is a three-dimensional diagram of the enclosure, and the lower portion is a front view of the thin metal plate that is set in the middle of the enclosure to divide it into two equal parts. The enclosure is assumed to be a cuboid box with the size of 2.3cm $\times$ 20cm $\times$ 32cm. Three narrow slots of 10cm length and 0.1cm width are cut on the thin metal plate. A current source along x-direction with a Gaussian-pulse temporal variation is placed at the center of the lower part of the enclosure. The observation point is set at the central point of upper part of the enclosure. To resolve the fine-scale geometric detail of the narrow slots in the x-direction, we choose the spatial grid sizes Δx =0.02cm and $\Delta y = \Delta z$ =1cm. Thus, the total mesh dimensions are 115 $\times$ 20 $\times$ 32 cells in the x- , y- and z-directions, respectively. For this problem, the conventional PITD method requires over 780 GB of memory, which cannot be afforded on personal computers. However, the SS-PITD method only takes up about 2.2MB of memory. Fig.3 shows the waveforms of E_x in time domain simulated by the SS-PITD method and the conventional FDTD method, respectively, with CFLN=0.5 for the FDTD and CFLN=6 for

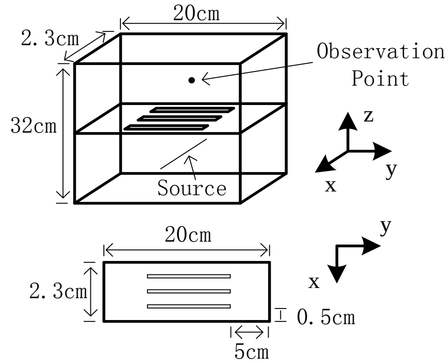

Fig.2 Enclosure with a thin metal plate

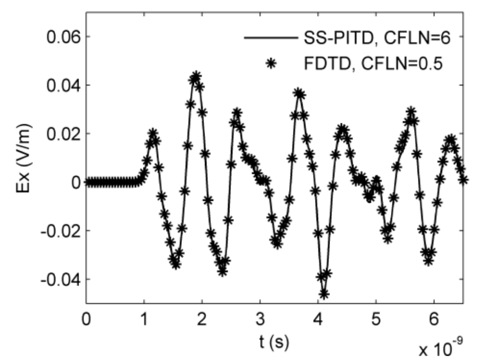

Fig.3 Waveforms of E_x excited by Gaussian pulse

the SS-PITD. It can be seen that the results are in quite good agreement.

To show that the proposed method takes up less memory than conventional PITD and achieves less dispersion error than the ADI-FDTD method, the resonant frequency of a simple 3-D air-filled cavity discretized by $8 \times 8 \times 8$ uniform grid cells of size 2 cm is calculated. It can be seen from Table I that, although the SS-PITD method is not in the same accuracy with the PITD method, it can be more memory efficient than the PITD method and more accurate than the ADI-FDTD method.

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