# Adaptive Discontinuous Galerkin Method for Transient Analysis of Eddy Current Fields in High-speed Rotating Solid Rotors

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*Abstract*—**For transient analysis of eddy-current fields in solid rotors which have invariant material properties along its motion direction, it is convenient to use the Eulerian formulation where only a fixed mesh is needed when modeling motion effect. However, a convection-diffusion equation has to be solved numerically instead of solving the pure diffusion equation in the Lagrangrian description of motion. Furthermore, when the rotor rotates at high-speeds, the eddy-current fields usually present sharp transition layers and also evolve with time, which makes it difficult to solve these fields accurately. To obtain high-resolution numerical solution efficiently, an adaptive discontinuous Galerkin method is adopted and numerical examples are given to showcase the accuracy and effectiveness of the proposed method.**

*Index Terms*—**Adaptive mesh, eddy-current magnetic field, discontinuous Galerkin, high-speed rotating.**

## I. Introduction

Solid rotors are commonly encountered in induction motors and magnetic brakes [1-2]. For these rotors of cylindrical shape, the material configuration and property of the rotor are invariably constant along the rotational direction. If the Lagrangian formulation is used to model rotational movement, special treatments must be paid to the meshing and matching boundary condition between the stator and the rotor of the devices. To ensure the accuracy of the numerical solution of the magnetic fields between the stator and the rotor, recently in [3] a novel slave-master technique with more slave modes than the master nodes on the sliding surface is proposed.

To avoid the difficulties when using multiple meshes to model motion in the Lagrangian formulation, it is preferable to use the Eulerian formulation for the modeling of eddy-current phenomena including motion effects. In the Eulerian description of motion, the eddy-current equation for these devices containing solid rotors expressed in time-domain is governed by [1]

$$\sigma\frac{\partial A}{\partial t}+\sigma\vec{v}\cdot\nabla A-\frac{\partial}{\partial x}(\nu\frac{\partial A}{\partial x})-\frac{\partial}{\partial y}(\nu\frac{\partial A}{\partial y})=J_s\,,t\in(0,T_f\,],\quad(1)$$

where $\partial\Omega$ is the boundary of the problem domain $\Omega$ ; $T_f$ is the stopping time of the analysis; $A$ is the axial component of the magnetic vector potential; $\sigma$ is the electric conductivity of the conductor; $\vec{v}$ is the velocity of the rotor; $\nu$ is the magnetic reluctivity; the excitation term $J_s$ is the current density of the applied source.

When the rotor rotates at high speeds, in the conducting region with positive conductivity, the governing equation is a convection-diffusion equation with dominated convection coefficient. It is well-known that the solutions of this type of equations usually contain sharp and narrow transition layers which may also evolving with time [4]. To get high-resolution numerical solution without using uniformly refined meshes which is too computationally time-consuming, one choice is to track the sharp layers by concentrating the mesh nodes around thin layers and dynamically reposition the mesh nodes with time. This is usually referred as the moving mesh method [5]. However, an extra governing equation named the moving mesh partial differential equation (MMPDE) to determine the motion of the mesh nodes has to be solved besides the physical equation concerned. In [6] the adaptive mesh finite element method, which allows for adaptive mesh refinement as well as mesh coarsening, for transient magnetic field analysis is adopted, which is however complicated to implement.

Recently, the discontinuous Galerkin (DG) method has been successfully applied to convection-dominated and other fluid flow problems [7-10]. The DG method can be viewed as an extension of the finite volume method and it allows flexible meshes containing hanging nodes since no inter-element continuity of the basis function is required in the DG method. This makes the *h*-adaptive refinement and coarsening processes much easier to realize. Hence the method is very flexible compared with traditional finite element method (FEM) when performing mesh refinement and de-refinement, as can be seen in Fig. 1.

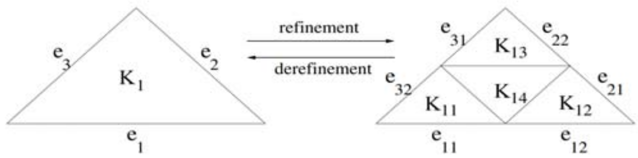


Fig. 1. A simple example illustrating the refinement and de-refinement operations.

In [7, 8], the DG method is used to solve some steady-state fluid flow problems using nonconforming meshes containing hanging nodes. In [9] the adaptive DG method was successfully applied to a two-phase flow problem. In this paper an adaptive discontinuous Galerkin (DG) method is proposed where the mesh is dynamically refined or coarsened according to the variations of the numerical solution. Numerical examples are also given to show the accuracy and effectiveness of the method.

## II. Discontinuous Galerkin Scheme for Eddy-current Equation

In this section, the formulation of the DG scheme is fully described. For convenience of discussion and without loss of generality, the following equation is taken as a model problem to illustrate the DG scheme:

$$\sigma\partial_t A + \nabla\cdot(\vec{f}(A) - \nu(|\nabla\times A|)\nabla A) = J_s,\ (\vec{x},t)\in\Omega\times(0,T_f]\ . \quad (2)$$

The definition of the DG scheme is following the same way given in [11]. Let $A_h$ be the DG approximation of the unknown magnetic potential $A$ in the space $V_h$, then multiply (2) by an arbitrary test function $v_h$ in the same space $V_h$ and integrate by parts, the semi-discrete DG formulation can be obtained as follows

$$\left(\sigma\frac{\partial A_h}{\partial t}, v_h\right)_\Omega + a_\varepsilon(A_h(t), v_h) + b(A_h(t), v_h) = L(A_h(t), v_h)\ , \quad (3)$$

where the definitions of the operators $a_\varepsilon(u,w)$, $b(u,w)$ and $L(u,w)$ are given by

$$a_\varepsilon(u,w) = \sum_{K\in T_h}\int_K \nu\nabla u\cdot\nabla w dV + \sum_{e\in E_h}\frac{m_0}{|e|^{n_0}}\int_e [[u]][[w]]ds - \sum_{e\in E_h}\int_e\{\{\nu\nabla u\}\}[[w]]ds + \varepsilon\sum_{e\in E_h}\int_e\{\{\nu\nabla w\}\}[[u]]ds\ , \quad (4)$$

$$b(u,w) = -\sum_{K\in T_h}\int_K \vec{f}(u)\cdot\nabla w dV + \sum_{e\in E_h}\int_e \hat{u}[[w]]ds\ , \quad (5)$$

$$L(u,w) = \int_\Omega J_s(u)\cdot w dV\ , \quad (6)$$

where $|e|$ is the length of the edge $e$, $\hat{u}$ is the upwind convective flux, $m_0$=1 and $n_0$=1 are stabilization parameters. In this paper the parameter $\varepsilon$ is set to be 1 and the resultant DG scheme is actually a non-symmetric interior penalty Galerkin method.

The nonlinear reluctivity in the DG formulation can be solved using traditional Newton-Raphson iteration method. At the starting time $t$=0, the initial value is projected onto the DG space $V_h$. For time discretization of (3), the backward Euler scheme is used in this paper.

## III. Numerical Example

In the numerical example, the adaptive DG method is applied to the numerical solution of the TEAM workshop problems 30A and 30B [12].

Select different rotation speeds, the magnetic fields are computed and the torque is also calculated to $T_f$=100ms for both problems. The torque errors of the DG method ($P^2$ DG basis functions are used and about 3700 nodes in the mesh) between the analytical values are shown in Fig. 2 and Fig. 3, respectively. For comparison, the results given in [12] using the traditional FEM are used as reference solutions. Once can clearly see that when the rotor is rotating at high speeds, the torque calculated by using the adaptive DG method is more accurate than using the FEM.

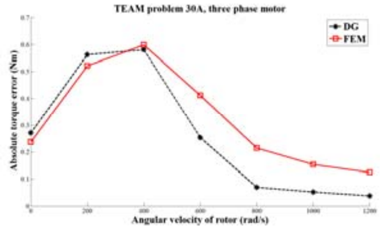


Fig. 2. Torque errors of the three-phase motor of the TEAM workshop problem 30A using DG and FEM for different rotor speeds.

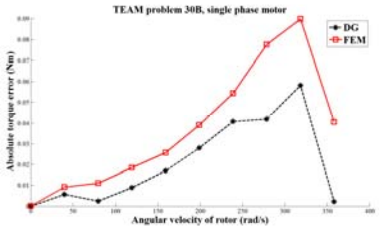


Fig. 3. Torque errors of the single-phase motor of the TEAM workshop problem 30B using DG and FEM for different rotor speeds.

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