The Natural Element Method Applied to Solve Electromagnetic Scattering Problem

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Abstract — In this paper, the natural element method with the traditional first order absorbing boundary condition (NEM-ABC) is applied to solve the classical electromagnetic scattering problem. The result is compared with the well known FEM-ABC and shows the applicability of the method and its relevance.

I. INTRODUCTION

The scattering problem occurs when an object is struck by an electromagnetic wave. The numerical solution of this problem is a topic of great interest in sciences and many engineering areas [1]. Traditional methods as the moment method and finite element method (FEM) has been applied in its solution but they often require an additional remeshing step to obtain accurate results when solving domain with edge or inverse problems [2]. Lately, meshless methods have been applied to solve boundary value problems in many areas including electromagnetics [2, 3]. Even, the Element Free Galerkin Method was already successfully applied to solve scattering problem [3].

The meshless methods give interesting responses to difficulties encountered in FEM such as remeshing noise. Although overcome difficulties encountered in FEM, these methods generally have a difficulties to take account the essential boundary conditions and discontinuities of the medium [5]. To eliminate these difficulties, the natural element method (NEM) was proposed [6]. The NEM approach is a halfway between the FEM and the other meshless methods. It is based on the Voronoi diagram and the natural neighbors. The main interest in NEM lies in its interpolation property that allows enforcing essential boundary conditions in an easy way, as it is with the FEM [6]. Also, this method retains the natural capacity of treating heterogeneous domain and present similar numerical behavior with a better convergence compared to FEM in some cases [7].

The aim of this paper is to apply the NEM to solve scattering problems. To limit the domain, an absorbing boundary condition of first order is applied. Thus, the proposed approach is denominated NEM-ABC. Since the main interest is evaluated NEM-ABC, the attention is restricted to 2D problems. A plane transverse magnetic wave is considered as an excitation. The results are compared with traditional FEM-ABC observing an excellent agreement between them.

II. THE NATURAL ELEMENT METHOD (NEM)

The natural element method uses the concept of natural neighbors. It is based on the construction of Voronoï diagram on a cloud of nodes. This diagram subdivides the studied domain into a set of polygons. Each polygon defines the natural neighbors of the node in its center. The Delaunay triangulation, which is the dual of the Voronoï diagram, is constructed by connecting the nodes whose Voronoï cells have common boundaries.

The interpolation of a point x can be obtained by introducing a fictitious point in the Delaunay triangulation to define its natural neighbors and calculating the value of the corresponding shape function.

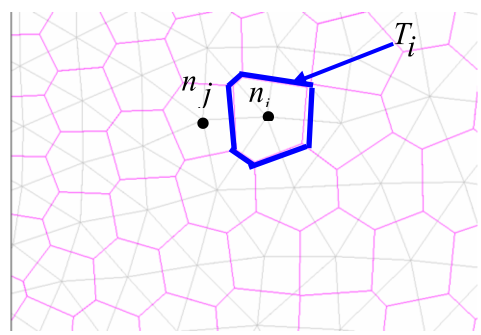

Fig. 1. Representation of Voronoï diagram (pink and blue color) and associated Delaunay triangulation (gray color)

Based on the Voronoï diagram, a natural element shape function can be calculated. In the literature, several formulas are used to calculate this shape function, but basically, the analogy is strong with the barycentric coordinates in the Delaunay mesh. Among the most used, are the functions of Sibson. It is based on the ratio of two surfaces linked to Voronoï cells [6].

Fig. 2. depicts the Sibson shape function related to a node far from the boundary of the domain. It is an interpolating function. Its support is given by the union of the circumscribed circles to the Delaunay triangles passing through this node. On the borders of the study domain, these functions become linear allowing a natural coupling to the FEM shape functions.

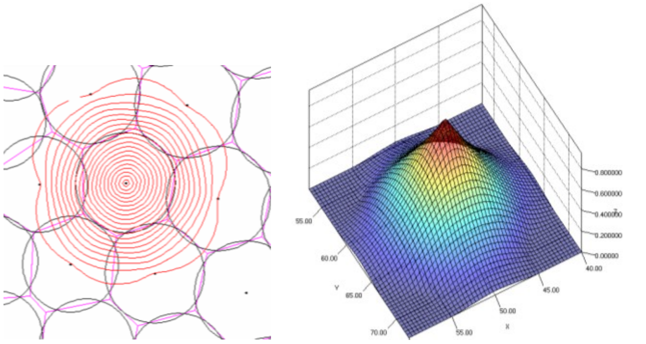

Fig. 2. (a) Support for NEM shape function and (b) Representation of the shape function at the middle domain.

III. PROBLEM FORMULATION

Consider the two-dimensional scattering problem due to non-homogeneous obstacles which properties are uniform along its infinite axis (z-axis), as illustrate in Fig. 3. To formulate this problem, a transverse magnetic TMz is assumed and the incident field is defined as:

$$E^i = e^{jk_0\left(x\cos\theta^i + y\sin\theta^i\right)}\ \hat{z} \qquad (1)$$

In (1) $k_0=2\pi/\lambda$ and θ^i is the incident electromagnetic fields angle relative to the given coordinate system.

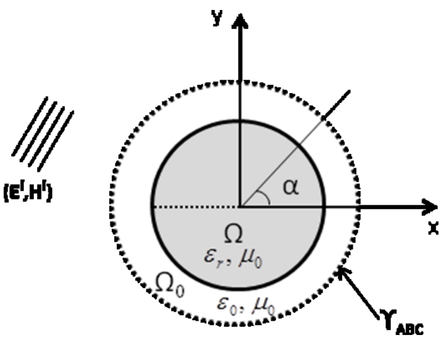

Fig. 3. Dielectric cylinder illuminated by a plane wave.

In Fig. 3, the transversal section of the scattered cylinder (the Ω domain) is shown in grey color. The dotted cylinder encloses the portion of the free space, Ω_0, necessary to the field computation. $\mathbf{E}^i$ and $\mathbf{H}^i$ are the incident electric and magnetic fields and γ_{ABC} is the boundary of the free space where the ABC condition is imposed. The weak form for the described problem can be written in as following [2]:

$$\int_{\Omega}\left[\nabla w\cdot\left(\mu_r^{-1}\nabla E_z\right)-k_0^2\,\varepsilon_r w E_z\right]d\Omega\ + \\ -\int_{\gamma_{ABC}}\mu_r^{-1}w\frac{\partial E_z}{\partial n}\,d\gamma_{ABC}=0 \qquad (2)$$

where E_z is the total electric field, w is the test function, μ_r and ε_r are, respectively, the relatives permeability and permittivity.

IV. RESULTS

To validate the NEM-ABC formulation, a computational code is implemented based on the discussed formulation. To show its effectiveness, the numerical results are compared with the analytical solution for a dielectric cylinder centered in the axis origin with circular cross-section of 0.3λ radius. The external circular boundary where the ABC of first order is applied has radius 0.75λ. Also, λ=1 and the number of unknowns are 844. Fig. 4 shows the FEM-ABC, the NEM-ABC and the exact solutions for an incident TMz wave with $\theta^i = 180^o$. These solutions are taken at the half upper dielectric surface for α ranging from 0^o to 180^o degrees. One can see from these results, a very good agreement between them which validate the NEM in scattering application domain. The analysis of these solutions has demonstrated that for the same number of unknown the NEM-ABC is quite accurate as FEM-ABC.

In the full paper, more details of the NEM and its implementation will be discussed. Other applications will be detailed and performed.

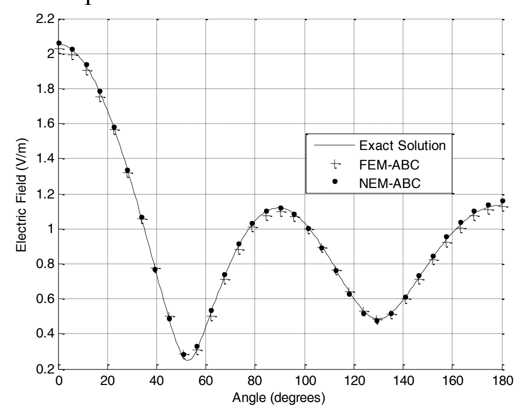

Fig. 4. Absolute value of the total electric field over the half upper semicircle of the dielectric surface.

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