Finite Element Analysis of Three-Phase Three-Limb Power Transformers under DC Bias

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***Abstract*—A finite element method for the analysis of three-phase, three-limb power transformers under DC bias is presented. The phase voltages and the DC components of the phase currents are assumed to be given. Using parallel algorithms, the steady state periodic solution is obtained without stepping through the transients using a fixed point method to solve the nonlinear equations. A novel technique to obtain the starting solution for the fixed point iterations to ensure fast convergence is introduced. Further methods to accelerate the solution are investigated.**

***Index Terms*—Power transformers, Finite element methods, Nonlinear equations, Stationary state, Parallel algorithms**

I. Introduction

A direct current (DC) bias in the windings of transformers results in strong saturation of the transformer core and this makes the field analysis problem extremely nonlinear. At the same time, the transients are of no interest, it is the steady state, periodic solution which is required to be computed. Such problems can be treated in the frequency domain using the harmonic balance method [1], [2], [3], in the time domain by means of the time periodic finite element method [4], [5] or in the discrete time domain [6].

The peculiarity of DC bias power transformer problems is that there are two exciting quantities: on the one hand, the sinusoidal voltage of the transformer windings is given and, in addition, the DC component of the winding current is also specified. This latter quantity gives rise to a DC bias in the flux and is hence not determined by the voltage, the time derivative of flux.

One possibility to overcome the problem of unusual excitations is to determine the waveform of the magnetizing current from simplified magnetic models [7]. However, in the case of three-phase transformers this is much more complicated than for single-phase ones (see [8]). In addition, no simple magnetic models for three-phase transformers without return legs, i.e. those with three-limb cores are available, since the DC flux must leave the core even at moderate saturation levels. This means that the magnetic models for the determination of the magnetizing current must include the tank and any magnetic tank shielding as well and are hence inherently complex.

An alternative method to treat DC bias problems with given voltage is the application of a special variation of the $\mathbf{T},\Phi$-Φ formulation in conjunction with a discrete time domain approach to get the steady state periodic solution and realized by means of fixed point iterations as introduced in [9]. However, it turns out that in case of three-phase, three-limb transformers the method requires a prohibitively high number of iterations to simultaneously obtain the field solution corresponding to the prescribed voltage and the magnetizing current waveform with the specified DC component. The aim of this paper is to solve this problem by enhancing the method of [9] to suit the peculiarities of three-phase, three-limb transformer problems with DC bias. This is achieved by introducing a technique to obtain a starting field solution and current waveform close enough to the correct results to ensure fast convergence of the method. Additionally, an alternative convergence acceleration technique will also be investigated.

II. The Model Investigated

A quarter model of a typical three-phase, three-limb power transformer is shown in Fig. 1.

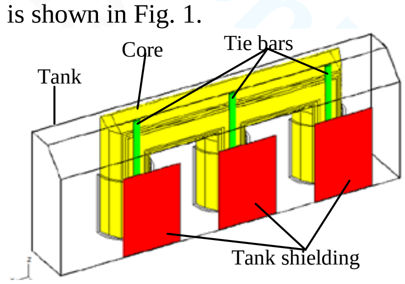

Fig. 1. Quarter model of a threephase, three-limb power transformer showing the tank, the core, the tie bars, the tank shielding and one winding each limb

The tank and the tie bars are made of massive steel and carry eddy currents. The core and the tank shielding are laminated and are assumed to be free of eddy currents. One phase winding excited by a given sinusoidal voltage is assumed around each limb carrying the magnetizing current with a prescribed DC component. The ferromagnetic materials of the tank, tie bars, core and tank shielding are nonlinear.

As described in [9], the application of the $\mathbf{T},\Phi$-Φ Galerkin formulation to a finite element discretization of the model leads to the system of ordinary differential equations

$$\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{S}(\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \mathbf{V}(\mu) & \mathbf{g}(\mu) & \mathbf{h}(\mu) \\ \mathbf{g}^{\mathrm{T}}(\mu) & \mathbf{M}(\mu) & \mathbf{G}(\mu) \\ \mathbf{h}^{\mathrm{T}}(\mu) & \mathbf{G}^{\mathrm{T}}(\mu) & \mathbf{L}(\mu) \end{bmatrix}\right)\begin{bmatrix} i \\ \mathbf{T}_h \\ \Phi_h \end{bmatrix} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} \qquad (1)$$

where the matrices on the left hand side are explained in [9]. The subscript h denotes the finite element approximations of the current vector potential $\mathbf{T}$ and the scalar potential Φ, i is the 3-vector of the winding currents and u is the 3-vector of the given voltages.

The fixed point method of [9] converges very slowly to the solution if the starting values for $\mathbf{T}_h$, Φ_h and i in the nonlinear iteration process are chosen as zero.

III. Determination of a Starting Solution

Let us simplify the model by neglecting the eddy currents, i.e. setting $\mathbf{T}$=0 in the tank and the tie bars. The starting solution for Φ_h and i will be obtained from the resulting static magnetic model. The flux linkages of the windings are the time integrals of the given voltages and are hence determined up to an unknown constant denoted by $\psi_{0,k}$ (k=1, 2, 3) for the three phases. The currents i_k of the windings can be computed from the given fluxes by applying the technique described in [10] to the magnetostatic Φ-formulation. This can be done for all N discrete time instants within one period T, resulting in the equations

$$\frac{1}{T}\int_0^T i_k\left(t,\psi_{0,1},\psi_{0,2},\psi_{0,3}\right)dt = I_{0,k}\left(\psi_{0,1},\psi_{0,2},\psi_{0,3}\right),\ k=1,\ 2,\ 3 \quad (2)$$

where $I_{0,k}$ are the given DC components of the currents. The integration can be computed numerically from the N discrete time values.

To evaluate the left hand sides of the three nonlinear equations in (2), N magnetostatic analyses have to be carried out with the flux time functions determined by the given voltages and by $\psi_{0,k}$ (k=1, 2, 3). These analyses are independent of each other and can be done parallel.

The system of nonlinear equations (2) is solved by a secant method instead of a Newton-approach to minimize the number of necessary evaluations of the left hand sides of (2). Once the solution is obtained, the time functions of the currents i_k as well as the time functions of the scalar potential values Φ_h over one period are available and can be used as starting values for the nonlinear iterations for solving (1).

IV. An Alternative Accelerating Technique

The solution process can be further accelerated by disregarding the tie bars in the model of Fig. 1 when computing the starting solution and using the current time function obtained as the excitation of the nonlinear periodic eddy current problem. This conventional problem with current driven windings can be solved by the method of [2]. The fixed-point method turns out to converge fast in this case. The resulting time functions for $\mathbf{T}_h$ and Φ_h can now be used as starting values to solve (1) using the method of [9] to achieve the correction of $i_k(t)$ due to the eddy currents.

V. Numerical results

The finite element discretization of the problem in Fig. 1 has resulted in 649,346 degrees of freedom (time functions over one period). The corresponding magnetostatic problem with $\mathbf{T}$=0 everywhere has 412,771 degrees of freedom.

Solving the problem using the method of [9] with zero starting values has failed to converge in several weeks.

For the determination of the starting solution by the method of section III has taken 10 secant iterations to solve (2) in about 85 hours on a 12 core X5690, 3.47 GHz computer. Using the resulting starting values, 8 fixed-point iterations in about another 9 hours were sufficient to solve (1).

The magnetostatic model without the tie bars involves no more than 216,736 degrees of freedom. With this model, 8 secant iterations in 37 hours have been sufficient to get the current wave form. Subsequently, the full eddy current problem with 649,346 degrees of freedom excited by this current time function could be solved in 8 fixed-point iterations in about 10 hours. This means an acceleration of about 50%.

The time functions of the power losses in the tank and the tie bars over one period are compared in Fig. 2. The losses from the approximate current wave forms agree well with those obtained from the model taking account of the eddy currents when determining the current time function.

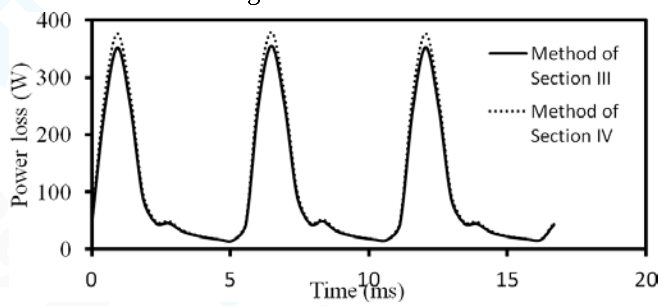

Fig. 2. Comparison of the time functions of the eddy current losses in the model (frequency is 60 Hz)

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