A Finite-Resistance-Network Based Method to Determine the Equivalent DC Resistance of a Conducting Region

R.B.B. Ovando-Martinez, C. Hernandez, M.A. Arjona
División de Estudios de Posgrado e Investigación, Instituto Tecnológico de la Laguna
Torreón Coah., 27000, México
ronat-13@hotmail.com, coni.hernandez@ieee.org, marjona@ieee.org

***Abstract*—A numerical method is presented to calculate the equivalent DC resistance of a device. The developed finite-resistance-network model was obtained from an electric circuit theory analysis and applying the principle of energy conservation. The finite-resistance-network is solved by using the nodal analysis from the electric circuit theory. The boundary conditions are current and/or voltage sources connected to the resistance network nodes. The developed model was used to calculate the equivalent DC resistance of a shunt. Afterwards, the results were compared with both, the finite element method and experimental data, and good results were achieved.**

***Index Terms*—DC resistance, electric circuits, finite element, nodal analysis.**

I. INTRODUCTION

The finite element method (FEM) has been applied in the design and improvement of electrical equipment. However, real engineering problems demands a high computing cost when they are analyzed by finite elements (FE). The equivalent circuit modeling is implemented instead of the analysis commonly made in electrical machinery modeling by FE [1],[2]. The equivalent circuit modeling does not require a high computing cost and it is solved by the standard circuit theories [3]. The equivalent circuit models are used to analyze the electrical behavior of a device without a field solution.

This paper proposes a finite-resistance-network based method to obtain the equivalent DC resistance of a conducting region where its electrical conductivity is known in advance. The proposed modeling estimates the equivalent DC resistance without experimental measurements as it is required by a real ohmmeter. The resulting finite-resistance-network is solved using the nodal analysis from electric circuit theory and the Thevenin resistance is calculated [4]. The computed Thevenin resistance represents the equivalent DC resistance of the conducting body. The developed technique was used to determine the equivalent DC resistance of a shunt that has an asymmetrical geometry. The obtained results were compared with both, the FEM and experimental data, and a good match was obtained.

II. FINITE RESISTANCE NETWORK MODEL

The equivalent circuit model of an electrical machine is obtained from experimental measurements of the voltage, current, speed and the electric power. A field distribution can be computed in a lumped parameter using the principle of energy conservation [5].

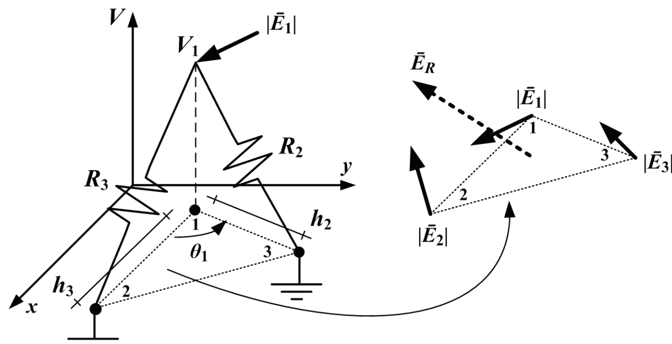

Fig. 1. Electric circuit representation of the nodal electric field energy.

The FE computes the electric field per element. The electric field $\bar{E}_R$ shown in Fig. 1 results from the addition of three independent electric field vectors $\bar{E}_1$, $\bar{E}_2$, $\bar{E}_3$ in the nodes. The electric field vector at a node is calculated as,

$$\bar{E}_n = \frac{V_n}{2S}\left[b_n \hat{a}_x + c_n \hat{a}_y\right] \tag{1}$$

where n is the node number, V_n the node voltage, $\hat{a}_x$-$\hat{a}_y$ unitary vectors in the x-y direction, respectively and b_n, c_n are obtained from de x and y node coordinates. The nodal electric power per node is calculated with the nodal electric field as,

$$W_n = \sigma\left|\bar{E}_n\right|^2 S \cdot h_z = \frac{\sigma \cdot h_z}{4S}\left[{b_n}^2 + {c_n}^2\right]\cdot {V_n}^2 \tag{2}$$

where W_n is the nodal electric power, S is the element area, σ the electric conductivity and h_z the length of the element in the z direction. In this paper, it is supposed that the nodal electric power is equivalent to the electric power dissipated in a resistor when is applied a voltage gradient between the nodes. The voltage gradient is measured from one node to the ground where the reference voltage is 0 by convention (see Fig. 1). In a triangular element, the nodal electric power (2) is dissipated by two resistors in parallel as it is shown in Fig. 1. The principle of energy conservation is used to compare the dissipated energy in the resulting parallel resistor from Fig. 1 with the nodal electric power in (2), this is obtained as,

$$W_n = \frac{\sigma \cdot h_z}{4S}\left[{b_n}^2 + {c_n}^2\right]\cdot {V_n}^2 = \left[\frac{1}{R_2} + \frac{1}{R_3}\right]\cdot {V_n}^2 \tag{3}$$

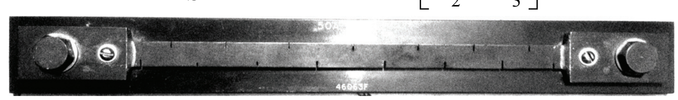

Fig. 2. Shunt used in the model.

The same process is applied to obtain a similar equation for the rest of the nodes. A system of three equations (as the one in (3)) and three unknowns (R_1,R_2,R_3) is obtained and solved to deduce the resistance equation per edge. As an example, the equation for the resistor R_1 was obtained as,

$$R_1 = \frac{-4S}{\sigma}\left[\frac{1}{h_2 h_3 \cos(\theta_1)}\right] \quad (4)$$

where h_2, h_3 is the edge length and θ_1 is the angle at node 1. The rest of resistors are obtained by permutation of the local element numbering in (4). Equation (4) is used to obtain the finite-resistance-network model of a discretized domain.

III. Examples

The finite-resistance-network modeling in (4) was programmed in C language. The model was applied to estimate the DC resistance of the shunt in Fig. 2. The shunt in Fig. 2 has an asymmetrical geometry. An ohmmeter can be easily used to measure the resistance of the shunt. However, the equivalent DC resistance can be calculated with the model in (4) where the electrical conductivity is known in advance. The electrical conductivity can be obtained from values in tables or experimental testing as the Electric Impedance Tomography (EIT) [6]. Figure 3(a) shows a graphical representation of the shunt shown in Fig. 2. The geometry of the shunt is discretized into a finite-resistance-network as in Fig. 3(b). Equation (4) is applied to calculate the resistors in the network. Afterwards, the nodal analysis is applied to obtain the system of equations in (5).

$$\lfloor G_{ij} \rfloor \lfloor V_j \rfloor = \lfloor I_i \rfloor \quad (5)$$

where G is an admittance matrix, V is the node voltage, I is the inlet/outlet node current and i,j denote numbers of nodes.

A. Boundary conditions

The system in (5) cannot be solved without imposing some boundary conditions. In a resistance, an electrical current can be injected in the electrodes A and B to determine the DC resistance, as it is shown in Fig. 3(b). In the system (5), the Dirichlet nodes correspond to the mesh nodes throughout the electrodes A and B. The vector of currents in (5) is modified to impose the inlet/outlet electrical current in the Dirichlet nodes. The resistors throughout an electrode are neglected in the admittance matrix to simulate a common electrical node. To avoid an infinite admittance matrix, the resistance of the resistors in an electrode is approached with a low value (1E-4 Ω).

B. Numerical and Experimental Results

The shunt in Fig. 2 was discretized using a 2D FE mesh with 7558 nodes and 14164 elements. The equivalent DC resistance of the shunt was estimated using (4) and (5). Table I shows the comparison results between the proposed model, the FEM and real experimental data. The test was performed with a ducter and applying a current magnitude of 1 A. The DC resistance obtained from the proposed model is closely to the results obtained from experimental data.

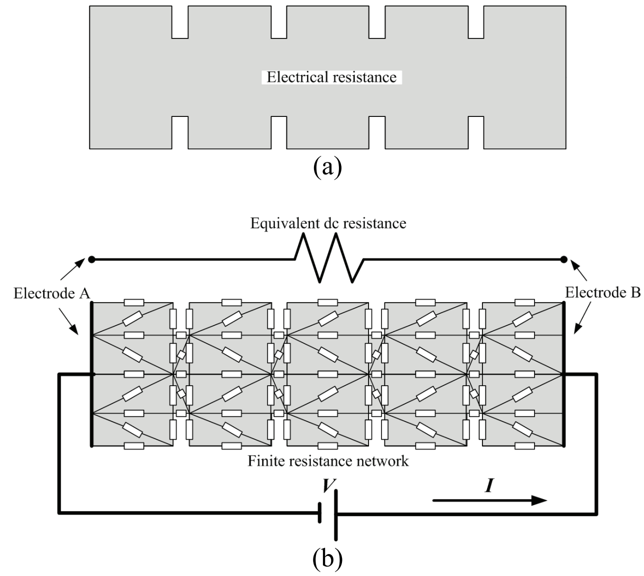

Fig. 3. Electrical resistance used in the model. (a) Electrical resistance accurate geometry for the shunt in Fig. 2. (b) A finite resistance network.

The variables V_A and V_B are the voltages in the electrodes A and B in the shunt, respectively. The estimated voltage gradient was closely to the FEM and the experimental results.

IV. Conclusions

A finite resistance network method was developed to estimate the equivalent DC resistance of a conducting region. The model was programmed in C language and used to calculate the equivalent DC resistance of a shunt. The results were compared with both, experimental data and the finite element, and good results were achieved. A more detailed analysis will be given in the full paper version.

TABLE I
COMPARISON RESULTS

	Model	FEM	Experimental
DC Resistance (mΩ)	4.989	4.982	5
V_A-V_B (mV)	4.989	5	5

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