Homogenization Techniques of Conductive and Non-magnetic Components Taking Account of Eddy Currents in Magnetic Field Analysis

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***Abstract*—In this paper, homogenization techniques for models composed of distributed conductive and non-magnetic components are investigated in linear ac steady-state eddy current problems. Two techniques are investigated: magnetostatic analysis with effective anisotropic complex permeability and eddy current analysis with modified anisotropic conductivity. In the latter technique, the method for determining modified anisotropic conductivity is proposed. It is shown that the former and latter techniques are suitable for the models, in which each conductive and non-magnetic component is insulated and connected, respectively.**

***Index Terms*—Eddy currents, electromagnetic shielding, finite element methods, homogenization, magnetic analysis.**

I. Introduction

A great effort toward the finite element modeling and a huge computation cost are required in the magnetic field analyses of models composed of distributed components, such as a building [1]. The homogenization techniques [1]–[7] are effective to circumvent these problems by modeling the distributed materials using a homogeneous body. The homogenization technique based on the energy conservation in magnetostatic analysis has already been proposed [4]. It has been applied to the analyses of magnetic disturbances of buildings [1] and the magnetic shielding performances of open-type magnetically shielded rooms composed of magnetic square cylinders [5]. On the other hand, the homogenization technique for a laminated core taking account of the eddy currents in the steel plates using effective permeabilities has also been proposed [6], [7]. However, the homogenization technique for the distributed conductive components taking account of eddy currents seems to be not established.

In this paper, the homogenization techniques for open-type shielding walls composed of the conductive and non-magnetic square cylinders with and without gaps taking account of eddy currents are investigated. Two homogenization techniques are examined for both models. One is the technique by the magnetostatic analysis with effective anisotropic complex permeability proposed in [6], [7]. The other is the technique by the eddy current analysis with modified anisotropic conductivity in which the method for determining the modified anisotropic conductivity is proposed. To clarify the suitable technique for each model, the shielding effects obtained using both homogenization techniques are compared with those obtained using the real model.

II. Model Description

Fig. 1 (a) shows the analyzed open-type shielding wall model composed of conductive and non-magnetic square cylinders. Only 1/8 of the whole model is analyzed due to symmetry. When a uniform ac flux density of B_{0x} = 1 T is applied to the shielding walls, the flux distributions on the line p-q shown in Fig. 1 (a) are calculated. Two models, in which cylinders are piled up with gaps G of 2 mm and without, are used. In the model without gaps, length L, width W, and thickness t of the cylinders, and frequency f are 20 mm, 100 mm, 4 mm, and 50 Hz, respectively. If the same dimensions and frequency are applied for the model with gaps, the shielding effect becomes too small. Therefore, L and f are changed to 100 mm and 500 Hz. The conductivity σ and relative permeability μ_s of the cylinders are 2×10^7 S/m and 1, respectively. Fig. 1 (b) shows the homogenized model, in which the shielding wall composed of cylinders is replaced to the homogenous body with the same outer dimension.

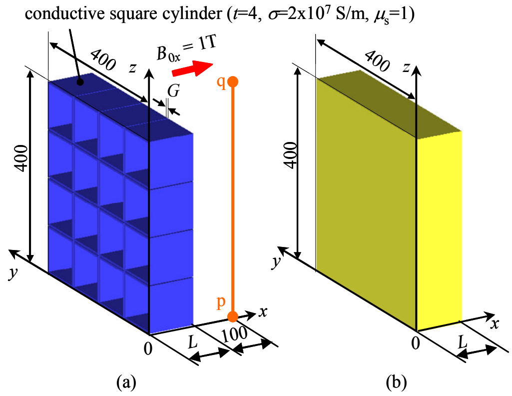

Fig. 1. Shielding wall model, (a) real model, (b) homogenized model.

III. Methods of Analysis

A. Magnetic Field Analysis

For the real model, linear ac steady-state eddy current analysis is performed using the 1st order edge finite element method with $\boldsymbol{A}$-ϕ method ($\boldsymbol{A}$: magnetic vector potential, ϕ: electric scalar potential) and the phasor method with complex variables. The fundamental equations are

$$\mathrm{rot}(\nu\, \mathrm{rot}\boldsymbol{A}^*) = -j\omega\boldsymbol{A}^* - \mathrm{grad}\phi^* \quad (1)$$

$$\mathrm{div}\{-\sigma(j\omega\boldsymbol{A}^* + \mathrm{grad}\phi^*)\} = 0 \quad (2)$$

where the superscript (*) indicates complex variables, ω and ν are the angular frequency and the reluctivity, respectively.

B. Homogenization Techniques

1) Effective Permeability

In this technique, the homogenized model is analyzed by using the magnetostatic field analysis neglecting the eddy currents. The effect of the eddy currents is considered using the effective anisotropic complex permeability μ_h*, moreover, unknown $\boldsymbol{A}$* is also complex to consider the phase delay by eddy current.

$$\mathrm{rot}(\nu_h{}^* \,\mathrm{rot}\boldsymbol{A}^*) = 0 \tag{3}$$

The x-component μ_{hx}* of μ_h* can be determined as follows. The linear ac steady-state eddy current analysis of the cell model in the uniform ac flux density B_{0x} , shown in Fig. 2, is carried out by using (1) and (2). The cell model is composed of one square cylinder surrounded by the air of 1 mm. The obtained magnetic field $H_{air,x}$ in the air gap does not include the compensation magnetic field of the eddy currents. Namely, $H_{air,x}$ means the magnetomotive force. Therefore, μ_{hx}* can be determined by using the following equation:

$$\mu_{hx}{}^* = B_{0x} / H_{air,x}{}^* \tag{4}$$

The other components can be obtained in the same way.

This technique is suitable for the wall model with gaps because it is the same as the homogenization technique of the laminated core proposed in [6], [7].

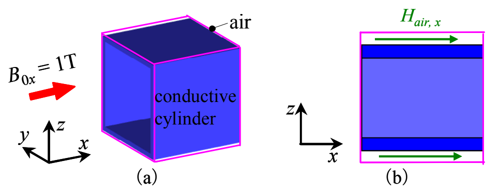

Fig. 2. Cell model for homogenization technique with effective permeability. (a) bird's eye view, (b) cross section at x-z plane.

2) Modified Conductivity

In this technique, the homogenized model is analyzed by using the eddy current analysis in (1) and (2) with the modified anisotropic conductivity σ_h.

σ_h is determined by equating the resistance of the homogenization model with that of the real cell model in each direction. For example, the z component σ_{hz} of σ_h is determined by using the following equation:

$$\sigma_{hz} = \sigma \cdot S_r / S_h \tag{5}$$

where S_r and S_h are the cross section areas of the conductor parts in the real and homogenized cell models shown in Fig. 3, respectively.

This technique is suitable for the model, in which the components are connected, because the eddy current paths coincide between the real and homogenized models.

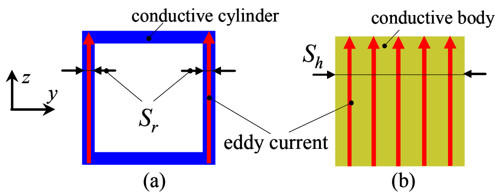

Fig. 3. Cell model for homogenization technique with modified conductivity, (a) real model, (b) homogenized model.

IV. RESULTS AND DISCUSSION

First, the flux distributions on the line p-q obtained using the real model with gaps and homogeneous one with μ_h* determined using (4) are compared in Fig. 4. The flux distribution obtained from the technique with μ_h (μ_{hsx}* =0.11–0.13i, μ_{hsy}* = μ_{hsz}* = 0.11–0.12i) is in good agreement with that obtained from the real model. For reference, the flux distribution obtained using the homogenization technique with σ_{hp} determined so that the flux density B_p at point p in the homogenized model coincides with that in the real model is also shown. In the technique with σ_{hp} large error occurs because the eddy current paths between the real and the homogenized models are much different due to the gaps.

Next, the flux distributions on the line p-q obtained using the real model without gaps and its homogenized one with σ_h determined using (5) are compared in Fig. 5. For reference, the result with σ_h' determined using the ratio of total volume of conductors instead of the ratio of the cross area in (5) is also shown. This figure shows that σ_h (σ_{hx}=3.1×10^6, σ_{hy}=σ_{hz} =1.6 ×10^6 S/m) should be used because eddy currents do not flow in the top and bottom planes as shown in Fig. 3.

It can be concluded that accurate magnetic field analyses of the models with complicatedly distributed conductors, in which each conductor is insulated and connected, are possible using suitable homogenization techniques with μ_h* and σ_h, respectively.

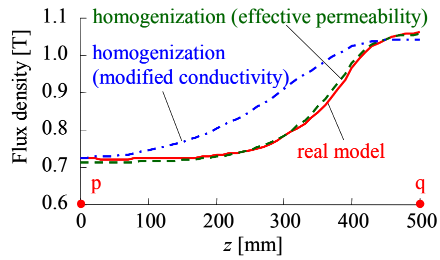

Fig. 4. Flux distributions on line p-q in model with gaps (L=100, G=2, f=500Hz).

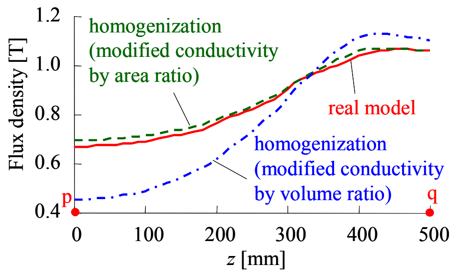

Fig. 5. Flux distributions on line p-q in model without gaps (L=20, G=0, f=50Hz).

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