Galerkin Finite Element Method for Calculating 3D Ion Flow Field

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***Abstract*—With HVDC system developing rapidly in China, the ion flow problems of the HVDC lines are concerned widely and some of which are 3D problem. A new 3D method together with Galerkin finite element method and Newton method is proposed in this paper. With subdivision of second-order tetrahedral elements, the numerical results and the analytic solution agree with each other very well and the new solution scheme proposed shows good convergence rate.**

***Index Terms*—Corona, HVDC transmission, Finite element methods, Newton method.**

I. Introduction

High voltage direct current (HVDC) transmission system has many advantages over HVAC transmission system for long-distance and large bulk power transmission [1]. So it has been developed rapidly in China. Two ±800kV ultra-high voltage direct current (UHVDC) transmission systems have been put into operation in China in 2010. Many UHVDC lines will be put into operation in the future.

HVDC transmission lines inevitably produce corona phenomenon and then the produced orientation moving space charge will significantly enhance electrostatic field produced by wire charge [1]. The electric field produced by space charge and wire charge is called ionized electric field. The ionized electric field under HVDC lines is an important problem of electromagnetic environment.

In China, the corridor of the HVDC lines is very complex that the HVDC lines may pass by rivers, buildings, forest. Then many of the ion flow field problems are considered as 3D problems. Many methods have been proposed to analyze the ion flow field of the HVDC transmission lines, but most of them focus on the 2D problems [2-6]. Based on Deutsch's assumption, that the ions in the space only change the intensity not the direction of the electric field, a 3D method for evaluating the ion flow field on the building is proposed [6]. But in the 2D ion flow field calculation, it's indicated that the Deutsch's assumption is not strictly true and may introduce some error [3].

The method coupled with Galerkin finite-element method and Newton iterative method has good convergence rate [7]. The method is easy to adopt high-order elements to obtain good precision. To solve 3D problem, the good convergence rate and high precision are extremely important. Though the method is only used to solve the 2D ion flow problem in [7], there is potentially probability for the method to be extended to solve 3D problem.

In this paper, the partial differential equations for ion flow field are discretized as nonlinear algebraic equations by using Galerkin finite-element method. To solve the nonlinear algebraic equations, Newton method in a new solution scheme is proposed. The new solution scheme shows good accuracy and convergence rate.

II. Methods

This paper focuses on the uniploar problem. For unipolar ion flow field under HVDC lines, the governing equations of the ion flow field are as follows [2]:

$$\nabla^2\varphi = -\rho/\varepsilon_0, \tag{1}$$

$$\boldsymbol{E} = -\nabla\varphi, \tag{2}$$

$$\nabla\cdot\boldsymbol{j} = 0, \tag{3}$$

$$\boldsymbol{j} = K\rho\boldsymbol{E}, \tag{4}$$

where φ is potential, E is electric field, ρ is charge density, ε_0 is permittivity of vacuum, $\boldsymbol{j}$ is ion current density and K is ion mobility.

Set $p = \rho/\varepsilon_0$ then (1)-(4) yield

$$\nabla^2\varphi = -p, \tag{5}$$

$$\nabla\varphi\cdot\nabla p - p^2 = 0. \tag{6}$$

The coupled partial differential system of (5) and (6) is nonlinear. To solve the problem, Galerkin finite-element method is adopted to discretize the differential system into an algebraic system. Tetrahedral element is adopted to divide the calculation domain. In order to accurately treat the curved surface, ten-node curved-side tetrahedral element is chosen [8].

The 3D solution scheme developed from [7] is noted by S1 and the solution scheme as follows is noted by S2.

Starting from an initial guess $p_{(0)}$, the basic iterative solution scheme is shown as follows:

i. $\varphi_{(m)}$ is obtained by using FEM from (5) with considering that $p_{(m)}$ is known.

ii. $p_{(m+1)}$ is obtained by using Galerkin FEM and a Newton iterative step from (6) with considering $p_{(m)}$ and $\varphi_{(m)}$ are known.

Now let's compare the two schemes through a numerical example of the ion flow in a homocentric sphere domain.

The radius of the inner sphere is 0.01 m and the radius of the outer sphere is 0.1 m. The potential of the inner surface is 50 kV and that of the outer surface is zero. For this structure, there is analytic solution of the ion flow field.

The numerical results by using S1, that by using S2 and the analytic result of the ion flow field are shown in Fig. 1. In Fig. 1(a) and Fig. 1(b), r is the distance from the centre of the sphere; In Fig. 1(a), ρ is charge density in the space; In Fig.

1(b), E is the electric field intensity. As shown in Fig. 1(a), the three values all coincide very well; in Fig. 1(b), the three values all coincide very well too. The two solution schemes S1, S2 are verified correct.

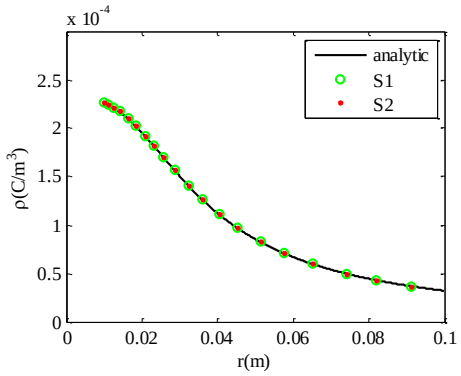
In Fig. 1, the difference of the values between S1 and S2 could not be found. In order to compare the values of the two methods carefully, the difference values are calculated. The maximal difference value of the charge density is about 1.0×10^{-14} C/m^3 and that of the electric field intensity is about 1.8×10^{-5} V/m. So the numerical results of the two schemes could be considered as the same value.

Now let's investigate the convergence rate of the two methods. The modulus values of the iterative correction amount by using the schemes S1, S2 are defined individually by C_m, D_m, where the subscript m indicates the m-th iteration. C_m, D_m are shown as follows:

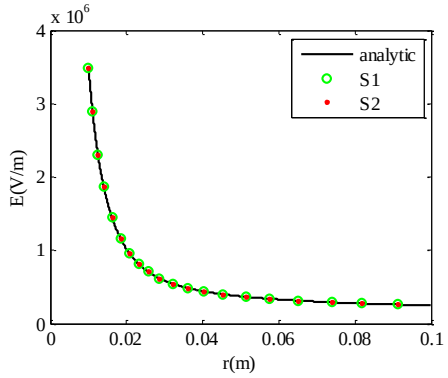
$$C_m = \left|\delta \mathbf{u}_{(m)}\right| = (\delta \mathbf{u}_{(m)} \cdot \delta \mathbf{u}_{(m)})^{1/2} \tag{7}$$

$$D_m = \left|\delta \mathbf{p}_{(m)}\right| = (\delta \mathbf{p}_{(m)} \cdot \delta \mathbf{p}_{(m)})^{1/2} \tag{8}$$

C_m, D_m varying with the number of the iterations m are shown in Fig. 2, where the vertical coordinate log(δ) indicates $\log(C_n/C_1)$ or $\log(D_n/D_1)$ for S1 or S2, and the horizontal ordinate is the number of the iterations. From Fig. 2, it could be found that the two methods have the similar convergence rate.



(a) The charge density



(b) The electric field intensity

Fig. 1 Comparison between numerical and analytic solutions of the ion flow field of the concentric sphere

The calculation precision and the convergence rate of the two schemes are similar. Next, let's analyze the calculation amount of the two schemes.

The number of the nodes is noted by *n*, then the order of **J**, the coefficient matrix for S1, is 2*n*. In the calculation, the storage of a $2n\times2n$ matrix is the major memory need for S1. While $\mathbf{J}^p$, the order of the coefficient matrix for S2, is *n*. So the major storage for S2 is two $n\times n$ matrixes. So the memory need of S2 is about 50% of S1.

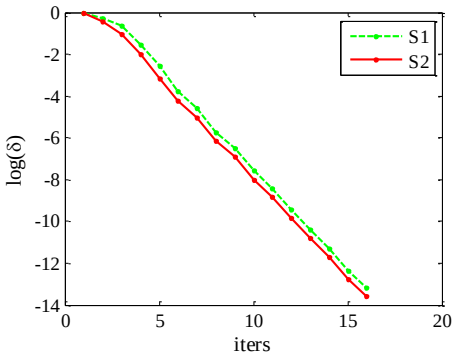


Fig. 2 Comparison of the convergence rate of the two schemes

In the calculation procedure, the solution of the nonsymmetric matrix equation is the major calculation amount. For large-scale matrix equation, the iterative algorithm is usually preferred to the direct algorithm. The condition number is an important target of the computation cost for iterative algorithm. The smaller the condition number is, the less the solution computation cost is. **J** and $\mathbf{J}^p$ are both nonsymmetric matrix. The condition number of **J** is 9.7×10^6, while the condition number of $\mathbf{J}^p$ is 1.3×10^5.

As above analysis, in memory need and solution cost, S2 is better than S1.

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