Modeling of Inductors Printed on Flexible Substrates Including the Edge Effect

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***Abstract*—Boundary element method has been included in software package for calculating the inductances of conductors printed on flexible substrates. The inductances are obtained by integration through the conductors. In this paper, the improvement of accuracy by including the edge effect at the corners of meander inductors is described. Current density is obtained from boundary conditions on the conductor's edges. The results for current density are validated by conformal mapping. The inductance calculation is validated by measurement results.**

***Index Terms*—inductors; flexible printed circuits; magnetostatics;**

I. Introduction

Recent developments in printed and flexible electronics have provided numerous new applications [1]. In our previous work, flexible inductors have been made and used for inductive position sensors and also as components in resonant circuits. The inductances of such, printed and usually spirally rolled, inductors has been calculated by in-house developed software package Provod. It has been shown in previous work that Provod's results are reliable and close to the measured values and inductances calculated by analytical expressions [2], [3]. However, it is possible to make the inductance calculation more accurate, especially in the case of meander conductors, by taking into account the edge effect at the corners.

In the Section II, the basic inductance calculation method is explained. In the Section III, the current density calculation method is given. In the Section IV, conformal mapping is used for verifying the obtained angles of current density vector. In the Section V, the inductances obtained with and without current density calculation are compared with measurement results. In the final section, a conclusion with comments about obtained results is given.

II. Inductance Calculation by Integration

It is possible to use finite element method for inductance calculation, but in order to avoid the problems with meshing due to very thin conductors and also the domain truncation error, an integral method for inductance calculation is used. Furthermore, assuming that there are no ferromagnetic materials in the studied problem (which is generally true in printed electronics), it is possible to obtain the inductances of printed conductors by double integration through the volume elements of all conductors in the system using the following expression [4]:

$$L=\frac{1}{I^2}\int \vec{J}\,\vec{A}\,dv \qquad (1)$$

where: J – current density vector in the element; A – vector potential at the center of the element; dv – volume of the element. The vector potential at a point is computed by integration through all the other elements in the problem [4]:

$$\vec{A}=\frac{\mu}{4\pi}\int \frac{\vec{J}}{R}\,dv \qquad (2)$$

where: R – distance between elements; μ – magnetic permeability (of vacuum, in our case). The integrals in (1) and (2) are calculated as sums over all the elements in all conductors, excluding the same element (R=0). In Provod, the printed conductor's geometry is entered as set of straight segments in u-v plane connected in series (see example in Fig. 1), each segment having uniform current density.

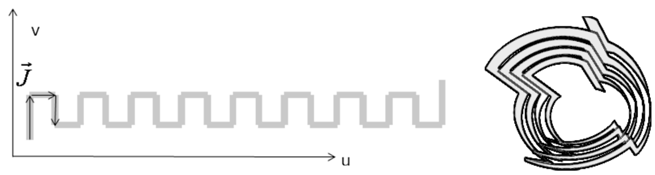

Fig. 1. (a) Meander conductor in u-v plane (b) Spirally rolled meander

The deformation of a flexible substrate is taken into account when three-dimensional coordinates of each element are calculated [3]. The u-v coordinates of elements obtained by the initial geometry input are transformed by the program into 3D coordinate system. The accuracy of the inductance calculation in Provod has already been confirmed in previously published papers [2], [3]. The accuracy has been tested for flat and curved substrates, such as spirally rolled and circular.

III. Current Density Calculation

For meander inductors, at the corners, the current density depends on 2D coordinates of an element. A conductor's corner with current flow and boundary conditions is shown in Fig.2.

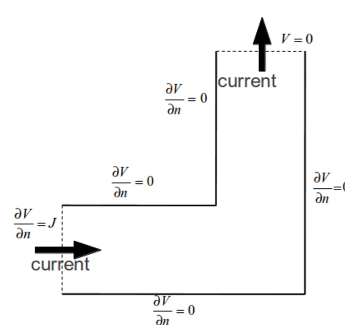

Fig. 2. Corner of a conductor

From Maxwell's equations for the case of DC conductors, Laplace's equation for the electric scalar potential is obtained [5]:

$$\nabla^2 V=0 \qquad (3)$$

The value of potential at a point inside a region can be expressed as an integral of boundary values of the potential and its derivative on the region's boundary Γ, as follows [5]:

$$V = \int w \frac{\partial V}{\partial n} d\Gamma - \int \frac{\partial w}{\partial n} V \, d\Gamma \tag{4}$$

$$w = \frac{1}{2\pi} \log\left(\frac{1}{r}\right) \tag{5}$$

where r is the distance from the point of integration to the point at which the potential is calculated. The current density is obtained as follows:

$$J = \sigma(-\nabla V) \tag{6}$$

From (4) and (6), with the electric conductivity σ adopted to be 1 (knowing that the problem will be scaled in order to obtain total current equal to 1):

$$\vec{J} = -\int (\nabla w) \frac{\partial V}{\partial n} d\Gamma + \int \nabla\left(\frac{\partial w}{\partial n}\right) V d\Gamma \tag{7}$$

The expression (7) has been applied in Provod. The values of V in (7) were obtained by (4), modified for points on boundary - boundary element method [5]. These values were calculated by a program written in C++.

IV. Comparison with Conformal Mapping

A conformal mapping can be used to transform a rectangular shape (trivial homogeneous field problem in z plane) into the corner shape (in w plane), see Fig.3. Using Schwarz-Christoffel transform [6], the derivatives of z and w with respect to auxiliary variable t (corresponding to a half plane) can both be written:

$$\frac{dz}{dt} = \frac{K_1}{\sqrt{t-t_0}\sqrt{t-t_1}\sqrt{t+t_0}\sqrt{t+t_1}} \tag{8}$$

$$\frac{dw}{dt} = \frac{K_2}{\sqrt{t}\sqrt{t-t_0}\sqrt{t-t_1}\sqrt{t+t_0}\sqrt{t+t_1}} \tag{9}$$

Equations (8) and (9) have been used to obtain the current direction on three different paths, A, B and C (at points with 0.5 mm spacing between them on either x or y axis). The paths A, B and C with corresponding points in w and z plane are presented in Fig.3 (a) and (b) respectively.

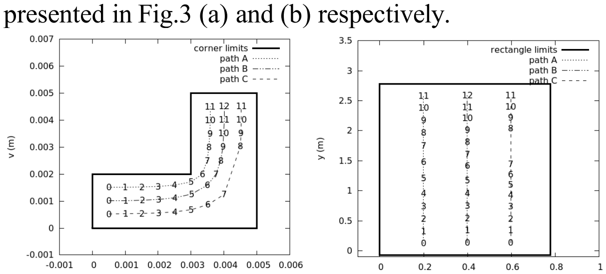

Fig. 3.(a) Corner in $w = u+jv$ plane (b) Rectangle in $z = x+jy$ plane

The angle of current density vector obtained by conformal mapping and in Provod are shown in Fig.4. The angle changes from 0 to pi/2, as expected. Very good agreement between conformal mapping and values calculated by Provod is obtained for all three observed paths.

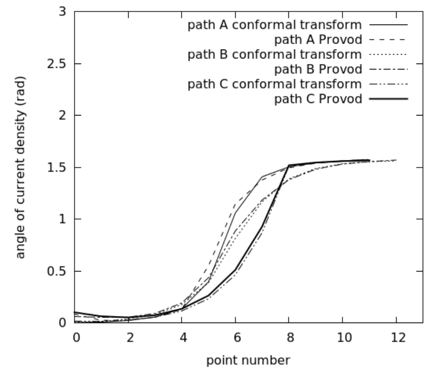

Fig. 4. Angle of current density on paths A, B and C

V. Comparison with Measurement Results

A meander-shaped inductor such as the one shown in Fig. 1(a), was printed on a flexible substrate which is then wound into spiral shape to obtain the conductor geometry shown in Fig.1(b). The spiral starts at radius 5 mm and 0.075 mm is added at each full turn, the vertical line segments are 10 mm long, all straight segments are 2 mm wide, 10 µm thick. The measured inductances (in plane and rolled) as well as calculated values with basic integration method and with edge effect included are given in Table I.

TABLE I
MEASURED AND CALCULATED INDUCTANCE OF A PRINTED MEANDER INDUCTOR

Inductor	Measured inductance [nH]	Inductance calculated without edge effect [nH]	Inductance calculated with edge effect [nH]
Meander in plane	190	202	191
Spirally rolled meander	490	514	488

By using the edge effect, an excellent agreement with experimentally obtained inductance values is obtained.

VI. Conclusion

The current density calculation at the corners of a meander inductor from boundary values of potential has been successfully applied for inductance calculation. The obtained angle of currant density matches closely the values found by conformal mapping. The inductances calculated for a meander conductor, in plane and wrapped, are in excellent agreement with measured values.

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