Spherical Harmonics Coefficients of All Magnetic Field Components Generated by Iron Piece

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***Abstract*—When a magnet generating highly homogeneous magnetic field is designed, a shimming is required. Usually, the shimming is performed for compensating the magnetic field only in the *z*-direction by an iron or a coil. Commonly the compensation is achieved for eliminating the coefficients of the spherical harmonics expansion of the magnetic field generated by the magnet. Some papers showed the coefficients of the spherical harmonics expansion in the *z*- direction for a passive shimming. However, recently, some magnets generate a tilted magnetic field, such as a magic angle rotating field NMR/MRI. Therefore, the coefficients of the spherical harmonics expansion in the *x*- and *y*-directions are presented in this paper.**

***Index Terms*—Magnetic analysis, magnetic resonance imaging, Nuclear magnetic resonance.**

I. Introduction

It is necessary to compensate a magnetic field around the center with a shimming when it is required to generate highly homogeneous magnetic field. The passive shimming, one of the shimming method, is to compensate the magnetic field using some pieces of iron [1]-[3]. Since the magnetic field homogeneity of a few PPM is required around the center of the magnet, the spherical harmonics expansion of the magnetic field is employed [4], [5]. The FEM and the BEM can hardly achieve such accuracy.

Commonly, a magnet for MRI or NMR is axially symmetric. Therefore, only the *z*-component of the magnetic moment of iron is considered so that only the *z*-component of the magnetic field is homogenized [4]. However, the open MRI magnet generates the axially asymmetric magnetic field. Therefore, all the components of the magnet moment of the iron piece have to be considered, hence the coefficients of the spherical harmonics expansion of *z*-direction magnetic field, that are generated by all the component of the magnetic moment, were presented in [5]. In addition, recently, a newly developed MRI magnet generates a tilted magnetic field for a magic angle rotating field MRI. Therefore, the coefficients of the spherical harmonics expansion of all the magnetic field component, that are generated by all the component of the magnetic moment, is strongly desired. Consequently, we calculated the spherical harmonics coefficients of all the magnetic field components for all the magnetic moment components. However, since there is no sufficient space in the paper, the spherical harmonics coefficients of only the *x*-component of the magnetic field for all the magnetic moment components are shown. We will show a computation result of a passive shimming to all the magnetic field components in the extended paper.

II. Calculation of Spherical Harmonics coefficients

The magnetic flux Φ generated by the magnetic moment $\vec{m}$ at point Q as shown in Fig. 1 is represented by

$$\Phi = -\frac{\vec{m}}{4\pi\mu_0}\cdot\nabla_{\mathrm{Q}}\left(\frac{1}{R}\right). \quad (1)$$

The symbols are shown in Fig. 1. Here, using the spherical harmonics function, $1/R$ is given by

$$\frac{1}{R} = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\varepsilon_m\frac{(n-m)!}{(n+m)!}\frac{r^n}{r_0^{n+1}}P_n^m(\cos\alpha)P_n^m(\cos\theta)\cos\left[m(\varphi-\phi)\right]. \quad (2)$$

The magnetic field $\vec{B}$ at point P is expressed as

$$\vec{B} = -\mu_0\nabla_{\mathrm{P}}\Phi. \quad (3)$$

The *x*-component of the magnetic field is expressed using (1) – (3) as

$$B_x = \sum_{n=0}^{\infty}\sum_{m=0}^{n} r^n P_n^m(\cos\theta)\left(C_x^{n,m}\cos m\varphi + D_x^{n,m}\sin m\varphi\right) \quad (4)$$

where

$$\begin{bmatrix} C_x^{n,m} \\ D_x^{n,m}\end{bmatrix} = \begin{bmatrix} C_{x,\rho}^{n,m} \\ D_{x,\rho}^{n,m}\end{bmatrix} + \begin{bmatrix} C_{x,\varphi}^{n,m} \\ D_{x,\varphi}^{n,m}\end{bmatrix} + \begin{bmatrix} C_{x,z}^{n,m} \\ D_{x,z}^{n,m}\end{bmatrix}, \quad (5)$$

$\vec{m} = (m_\rho, m_\phi, m_z)$ in the cylindrical coordinate, the size of a piece of iron is defined in Fig. 1(b), and the coefficients of (5) are shown in the last of the paper.

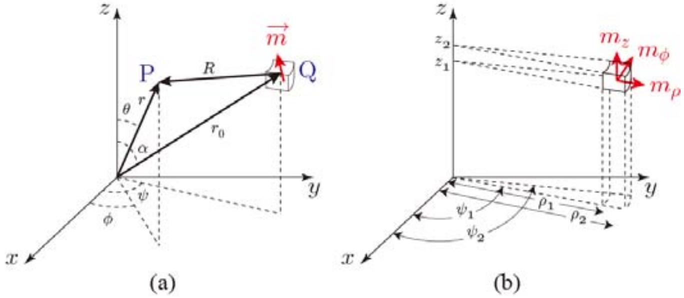

Fig. 1. (a) Magnetic moment $\vec{m}$ at point *Q* generates magnetic flux Φ at point P (b) Dimensions of a piece of iron.

III. Conclusion

We present the spherical harmonics coefficients of all the components of the magnetic field to all the components of an iron piece for compensating the magnetic field generated by the magic angle rotating field MRI magnet by means of the passive shimming.

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$$\begin{bmatrix} C_{x,\rho}^{n,m} \\ D_{x,\rho}^{n,m} \end{bmatrix} = \begin{cases} \begin{bmatrix} \frac{m_\rho}{8\pi}\left(W_{n+2}^{2} - (n+1)(n+2)W_{n+2}^{0}\right)\left(\sin\varphi_2 - \sin\varphi_1\right) \\ 0 \end{bmatrix} & \text{for } m=0, n\ge 0 \\ \frac{m_\rho}{16\pi}\left\{\frac{W_{n+2}^{3}}{n(n+1)} - W_{n+2}^{1}\right\}\begin{bmatrix} \sin 2\varphi_2 - \sin 2\varphi_1 \\ -\cos 2\varphi_2 + \cos 2\varphi_1 \end{bmatrix} - \begin{bmatrix} \frac{m_\rho}{4\pi} W_{n+2}^{1}\left(\varphi_2 - \varphi_1\right) \\ 0 \end{bmatrix} & \text{for } m=1, n\ge 1 \\ \frac{m_\rho}{8\pi}\frac{(n-m+2)!}{(n+m)!}\left\{\frac{(n-m+3)(n-m+4)}{m-1}W_{n+2}^{m-2}\begin{bmatrix} \sin(m-1)\varphi_2 - \sin(m-1)\varphi_1 \\ -\cos(m-1)\varphi_2 + \cos(m-1)\varphi_1 \end{bmatrix} \right. & \\ + \frac{W_{n+2}^{m+2}}{(n-m+1)(n-m+2)(m+1)}\begin{bmatrix} \sin(m+1)\varphi_2 - \sin(m+1)\varphi_1 \\ -\cos(m+1)\varphi_2 + \cos(m+1)\varphi_1 \end{bmatrix} & \\ \left. - \frac{2W_{n+2}^{m}}{m^2-1}\begin{bmatrix} m\left(\cos\varphi_2 \sin m\varphi_2 - \cos\varphi_1 \sin m\varphi_1\right) - \left(\sin\varphi_2 \cos m\varphi_2 - \sin\varphi_1 \cos m\varphi_1\right) \\ -m\left(\cos\varphi_2 \cos m\varphi_2 - \cos\varphi_1 \cos m\varphi_1\right) - \left(\sin\varphi_2 \sin m\varphi_2 - \sin\varphi_1 \sin m\varphi_1\right) \end{bmatrix}\right\} & \text{for } m\ge 2, n\ge 2 \end{cases}$$

$$\begin{bmatrix} C_{x,\phi}^{n,m} \\ D_{x,\phi}^{n,m} \end{bmatrix} = \begin{cases} \begin{bmatrix} -\frac{m_\phi}{8\pi}\left(W_{n+2}^{2} + (n+1)(n+2)W_{n+2}^{0}\right)\left(\cos\varphi_2 - \cos\varphi_1\right) \\ 0 \end{bmatrix} & \text{for } m=0, n\ge 0 \\ -\frac{m_\phi}{16\pi}\left\{\frac{W_{n+2}^{3}}{n(n+1)} + W_{n+2}^{1}\right\}\begin{bmatrix} \cos 2\varphi_2 - \cos 2\varphi_1 \\ \sin 2\varphi_2 - \sin 2\varphi_1 \end{bmatrix} & \text{for } m=1, n\ge 1 \\ \frac{m_\phi}{8\pi}\frac{(n-m+2)!}{(n+m)!}\left\{\frac{(n-m+3)(n-m+4)}{m-1}W_{n+2}^{m-2}\begin{bmatrix} \cos(m-1)\varphi_2 - \cos(m-1)\varphi_1 \\ \sin(m-1)\varphi_2 - \sin(m-1)\varphi_1 \end{bmatrix} \right. & \\ - \frac{W_{n+2}^{m+2}}{(n-m+1)(n-m+2)(m+1)}\begin{bmatrix} \cos(m+1)\varphi_2 - \cos(m+1)\varphi_1 \\ \sin(m+1)\varphi_2 - \sin(m+1)\varphi_1 \end{bmatrix} & \\ \left. + \frac{2W_{n+2}^{m}}{m^2-1}\begin{bmatrix} m\left(\sin\varphi_2 \sin m\varphi_2 - \sin\varphi_1 \sin m\varphi_1\right) + \left(\cos\varphi_2 \cos m\varphi_2 - \cos\varphi_1 \cos m\varphi_1\right) \\ -m\left(\sin\varphi_2 \cos m\varphi_2 - \sin\varphi_1 \cos m\varphi_1\right) + \left(\cos\varphi_2 \sin m\varphi_2 - \cos\varphi_1 \sin m\varphi_1\right) \end{bmatrix}\right\} & \text{for } m\ge 2, n\ge 2 \end{cases}$$

$$\begin{bmatrix} C_{x,z}^{n,m} \\ D_{x,z}^{n,m} \end{bmatrix} = \begin{cases} \begin{bmatrix} \frac{m_z}{4\pi}\left(n+1\right)W_{n+2}^{1}\left(\sin\varphi_2 - \sin\varphi_1\right) \\ 0 \end{bmatrix} & \text{for } m=0, n\ge 0 \\ \frac{m_z}{8\pi}\frac{W_{n+2}^{2}}{n+1}\begin{bmatrix} \sin 2\varphi_2 - \sin 2\varphi_1 \\ -\cos 2\varphi_2 + \cos 2\varphi_1 \end{bmatrix} - \begin{bmatrix} \frac{m_z}{4\pi}\left(n+2\right)W_{n+2}^{0}\left(\varphi_2 - \varphi_1\right) \\ 0 \end{bmatrix} & \text{for } m=1, n\ge 1 \\ \frac{m_z}{4\pi}\frac{(n-m+2)!}{(n+m)!}\left\{-\frac{n-m+3}{m-1}W_{n+2}^{m-1}\begin{bmatrix} \sin(m-1)\varphi_2 - \sin(m-1)\varphi_1 \\ -\cos(m-1)\varphi_2 + \cos(m-1)\varphi_1 \end{bmatrix} \right. & \\ \left. + \frac{W_{n+2}^{m+1}}{(n-m+2)(m+1)}\begin{bmatrix} \sin(m+1)\varphi_2 - \sin(m+1)\varphi_1 \\ -\cos(m+1)\varphi_2 + \cos(m+1)\varphi_1 \end{bmatrix}\right\} & \text{for } m\ge 2, n\ge 2 \end{cases}$$

$$W_n^m \equiv \int_{\rho_1}^{\rho_2}\int_{z_1}^{z_2} \frac{1}{r_0^{n+1}} P_n^m(\cos\alpha)\,\rho\,\mathrm{d}\rho\,\mathrm{d}z$$