A Novel Hybrid Method Combining DGTD and TDIE for Wire Antenna-Dielectric Interaction

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Abstract—This paper presents a hybrid method that effectively combines two versatile numerical methods - the discontinuous Galerkin time domain (DGTD) method and the time domain integration method (TDIE). The hybrid method is highly applicable to the coupling problem between arbitrarily-shaped thin-wire and dielectric structure with complex media compositions. The Huygens equivalence principle is employed to divide the original problem into two sub-problems: i) the region containing the thin-wires, which is calculated by using the TDIE method, and ii) the dielectric region modeled with unstructured grids and analyzing by the DGTD method. To validate the hybrid method, several numerical examples are presented, proving the proposed method a promising scheme for the coupling problem.

Index Terms—Electromagnetic coupling, numerical analysis, time domain analysis.

I. INTRODUCTION

Such electromagnetic problem as arbitrarily oriented thinwire antennas coupled with nearby inhomogeneous dielectric scatterers are widely encountered in wireless applications. In order to predict this phenomenon accurately and efficiently, many numerical methods have been used. The popular finite difference time-domain (FDTD) method [1] has advantage of simple implementation, but has staircase error when dealing with complex geometric features. Finite element (FE) method [2] can mitigate the staircase errors by employing unstructured grids, but it becomes source-consuming dealing with electrically small thin-wire strucutres. The method of moment (MoM) is good at resolving the radiation of thin-wire structure located in free space, but has difficulties to deal with inhomogeneous dielectric objects.

Hence, a single method is often unable to deal the complexity in geometry and media composition of abovementioned problem. Hybrid methods, which combine the desirable features of two or more different techniques, have been developed to analyzed complex electromagnetic problems. Bretones proposed the TDIE/FDTD method [3] and the FDTD/FETD/TDIE method [4] in 1998 and 2004 respectively. They both employ TDIE to handle the thin-wire radiation problems, meanwhile the inhomogeneous objects are analyzed by FDTD in the former one and FETD in the latter one. However, as mentioned before, the former method suffers from staircase error the latter method has high computational cost.

The discontinuous Gakerkin time-domian (DGTD) method [5], which combines the geometrical versatility of FE method with the explicit time-stepping of FV time-domain (FVTD) method, has advantages of handling arbitrarily shaped curved

objects than FDTD, and that of less computational cost than FETD. It exceeds FDTD in accuracy and FETD in efficiency, thus being very suitable to replace FDTD and/or FETD in dealing with this complex problem.

In this paper, we present a novel hybrid method bringing together the DGTD and the TDIE methods to analyze the coupled problems including thin-wire antennas and arbitrarily-shaped inhomogeneous bodies. As shown in Fig. 1, the TDIE is employed to analyze the thin-wire antennas in region R^{IE} , while the DGTD is used to solve the curved and inhomogeneous dielectrics in region R^{DG} with the absence of antennas. And the interaction of them is through the Huygens surface *S*. The hybrid DGTD-TDIE presented here integrates the desirable capabilities of the individual methods for resolving different parts of the problem for which they are the most suitable. It has the advantage to analyze more realistic complex problems and implement more simply than its hybrid counterparts.



Fig. 1. Hybrid DGTD-TDIE setup for wire antenna-dielectric interaction

II. DESCRIPTION OF THE HYBRID METHOD

The DGTD method in R^{DG} is the high-order nodal DGTD method and the TDIE in R^{IE} is to solve the electric field integral equations (EFIE) with linear spatial basis functions. Here, we only focus on the detailed hybrid time-stepping procedure:

Firstly, at arbitrary time step in region R^{IE} , as the feeding source and external scattering source are already known from the previous calculation, the currents on the thin-wire antenna can be easily obtained by solving EFIE. These currents lead to the equivalent sources on Huygens surface *S* which yields the same radiating fields outside S and null inside. The equivalent sources can be expressed as equivalent electromagnetic fields E^{IE} and H^{IE} . Next, the DGTD method is applied in the entire computational domain R^{DG} without the antenna in R^{IE} . Here, we directly inject the equivalent fields E^{IE} and H^{IE} in R^{DG} by revising the numerical fluxes [6] on *S*. The field outside *S* is the sum of the incident and the scattered fields scattered by the dielectric, while inside is only the scattered field. The scattered fields E^{DG} at certain points on the antenna can be evaluated by linear interpolation.

Finally, provided the scattered fields E^{DG} and the feeding source on the antenna at current time step, the E^{IE} and H^{IE} at next time step on S will be easily expected. Then, the recursive procedure can be repeated.



Fig. 2. Hybridization timing diagram

The above procedure assumes Δt^{DG} is equal to Δt^{IE} . Practically, as the DGTD and the TDIE methods are explicit and implicit in time domain respectively, Δt^{DG} could be much smaller than Δt^{IE} , viz. $\Delta t^{IE} = k \times \Delta t^{DG}$. To synchronize them, we simply perform one TDIE calculation followed by *k*-time DGTD routines, as shown in Fig. 2. During TDIE time step Δt^{IE} ([t_n, t_{n+1}]), The scattered field $E^{DG}(t_n)$ needed by TDIE routine is only calculated at the final Δt^{DG} of previous Δt^{IE} , and the source fields, $E^{IE}(t_n+i\Delta t^{DG})$ and $H^{IE}(t_n+i\Delta t^{DG})$, i=0...(k-1), needed by each DGTD routine is obtained by linear interpolation at specific time. This approach will lead to a balancing and stable hybridization technique regardless of the imbalance of the time steps.

III. NUMERICAL RESULTS

To verify the capability of the proposed hybrid DGTD/TDIE method, a straight thin-wire antenna located in the neighborhood of a perfectly electric conducting (PEC) plane is first presented, as shown in Fig. 3. The antenna is 0.2m in each arm and 2mm in radius. It is modeled with 10 segments and excited at its center by a 4-lightmeter width Gaussian pulse voltage source. The distance between the antenna and the PEC plane is 1m. The PEC plane is of $2m \times 2m$. And R^{DG} is of $5m \times 5m \times 5m$ with the PML, meshed by 6544 tetrahedrons. The comparable numerical results of E_x at point P by using hybrid DGTD-TDIE, TDIE (3600 faces) and DGTD (19687 grids) are also presented in Fig. 3. Great agreement could be observed between the hybrid method and the TDIE method. However the result of the DGTD method is not as good as the other two, despite the more refined mesh it used. A V-antenna radiating a dielectric sphere with relative permittivity $\varepsilon_r=2$ is further analyzed, as shown in Fig. 4. The V-antenna has an aperture of 60° and the sphere has radius of 0.2m. The distance between the center of the sphere and the antenna is 0.6m. The other parameters are the same like the first case. The results of E_x at point P of the hybrid and TDIE methods are also presented in Fig. 4. Although the results agree well at first, we could find obvious late time instability in the result of the TDIE method.

IV. CONCLUSIONS

A simple yet useful hybridization scheme combining DGTD and TDIE methods has been proposed and employed for the coupling problem between thin-wire antenna and dielectrics. It combines the advantages of the DGTD method in dealing with complex geometries and media compositions and that of TDIE method in treating arbitrarily oriented thin-wire structures. It offers an alternative to the other classic/hybrid method in evaluating transient solution of such problems.





Fig. 3. Amplitude of the x component of the total electric field at point P

Fig. 4. Amplitude of the x component of total electric field at point P

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