Efficient Implementation of the CFS-PML on Curved Two-Dimensional Domain

A. S. Moura[1], R. R. Saldanha[2], E. J. Silva[2], A. C. Lisboa[3] and W. G. Facco[4]
[1]Department of Economic Sciences, Federal University of Juiz de Fora, Brazil
[2]Department of Electrical Engineering, Federal University of Minas Gerais, Brazil
[3]ENACOM - Handcrafted Technologies, Brazil
[4]General Education - Federal Institute of Espirito Santo, Brazil,

***Abstract*—The time domain maxwell's equations are solved using discrete differential forms. The theory of discrete differential forms to the numerical solution of boundary value problems derived from the discretization of Maxwell's equations. For this class of problems, an unbounded domain must be truncated by an absorbing boundary to get a limited computational domain. The complex frequency shift perfectly matched layer (CFS-PML) truncates the computational domain by a reflectionless artificial layer which theoretically absorbs outgoing waves regardless of their frequency and angle of incidence. This article presents the formulation of a CFS-PML for differential forms in curved domains. It will be shown that the CFS-PML formulation on the differential forms framework is implemented through the directional incidence matrices. Also, curved domains are treated using the nearest neighbor. The performance in practice of the proposed technique is presented through some numerical simulations.**

***Index Terms*—Differential forms, absorbing boundary conditions, incidence matrix.**

I. Introduction

In applications to problems of electromagnetic wave, curved geometries are often faced and needed to be modeled in an appropriate way. In this context, we have also to consider the imposition of absorbing boundary conditions. For these applications, the original perfectly matched layer (PML), proposed by *Berenger* [1] must be modified in order to suit the more general form of domain truncation: it must suit curved geometries.

Another approach that has been investigated is the conformal PML (CPML), especially in the works *Kuzuoglu and Mittra* [2], and *Donderici and Teixeira* [3]. In the latter, a conformal PML is introduced to the finite element time-domain for the solution of Maxwell's equations in time domain.

Moura et al. [4] presents a new formulation to implement the Cartesian (CFS-PML) for domain truncation in 2-D directly applied to Maxwell's equations in differential forms. It is shown that the proposed method is highly absorptive to evanescent modes when computing the wave interaction of elongated structures or sharp corners. The impact of the CFS-PML parameters on the reflection error is investigated and optimal choices of these parameters are derived.

This work shows that the CFS-PML implemented using the first order discretization of Maxwell's equations [4] can extended to curved domains. The use in curved domains will be made by calculating the values of PML parameters within the PML region using the concept of nearest neighbor.

II. Formulation

Consider a simplicial mesh that represents a two-dimensional curved domain. Basically, to implement the curved PML layer we need a procedure to compute the distance from a point inside PML to its internal boundary.

Let D_p be the set of all points in the PML region which are nearest neighbors to point p of the boundary C. The parameter d on the entire D_p is the largest ρ on the points in D_p.

$$d = \max\{\rho_i, \rho_j, \rho_k\} \tag{1}$$

where ρ_i is the Euclidean distance from point i to point $p \in C$.

Fig.1(a) shows the calculation of ρ for an element of the PML region and Fig.1 (b) shows the behavior of the ratio ρ/d. Note that there is a smooth change from zero to one. By discretizing in time domain and applying techniques to calculate the convolution of recursive way, we obtain the following leap-frog scheme[4]

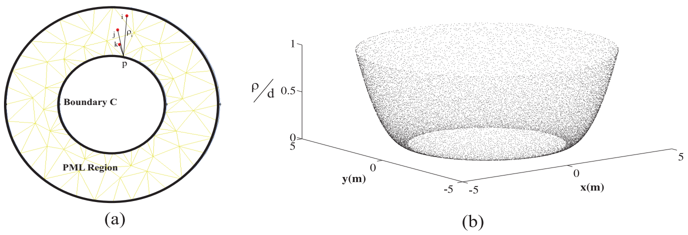

Figure 1: (a) Calculation of ρ for an element of the region PML. (b) Behavior of the ratio ρ/d.

$$\mathbf{b}^{n+\frac{1}{2}} = \mathbf{b}^{n-\frac{1}{2}} - \Delta t \mathbf{C}\left(\mathbf{K}_e^n + \mathbf{\Phi}^n\right) \tag{2}$$

$$\mathbf{N}_1 \mathbf{e}^{n+1} = \mathbf{M}_1(\epsilon)\,\mathbf{e}^n + \Delta t \mathbf{C}^T \mathbf{M}_2(\nu)\left(\mathbf{K}_b^{n+\frac{1}{2}} + \mathbf{\Psi}^{n+\frac{1}{2}}\right) \tag{3}$$

where

$$\mathbf{N}_1 = \left(\mathbf{M}_1(\epsilon) + \Delta t \mathbf{M}_1(\sigma)\right) \tag{4}$$

The inverse the matrix $\mathbf{N}_1$ can be approximated per a sparse matrix as shown in [5]. In (2) and (3), the vectors $\mathbf{\Phi}(t)$, $\mathbf{\Psi}(t)$, $\mathbf{K}_e(t)$, and $\mathbf{K}_b(t)$ are defined as follows

$$\mathbf{\Phi}(t) = [\phi_1(t), \cdots, \phi_{N_e}(t)] \tag{5}$$

$$\mathbf{\Psi}(t) = \left[\psi_1(t), \cdots, \psi_{N_f}(t)\right] \tag{6}$$

$$\mathbf{K}_e(t) = \left[e_1(t)/k_1, \cdots, e_{N_e}(t)/k_{N_e}\right] \tag{7}$$

$$\mathbf{K}_b(t) = \left[b_1(t)/k_1, \cdots, b_{N_f}(t)/k_{N_f}\right] \tag{8}$$

where their coordinates are defined in terms of the PML parameters, as follows

$$\phi_i(t) = \xi_i(t) * e_i(t) \tag{9}$$

$$\psi_j(t) = \xi_j(t) * b_j(t) \tag{10}$$

$$\xi_i(t) = -\frac{\sigma_i}{\epsilon_0 k_i^2} e^{-\left(\frac{\sigma_i + \alpha k_i}{\epsilon_0 k_i}\right)t} u(t) \tag{11}$$

The PML parameters in Eq.11 are determined by

$$k = 1 + (k_{\max} - 1)\left(\frac{\rho}{d}\right)^m \tag{12}$$

where $k_{\max}$ are the maximum values of k at the exterior boundary. The conductivity σ it is described as

$$\sigma = \sigma_{\max}\left(\frac{\rho}{d}\right)^m \tag{13}$$

where $\sigma_{\max}$ is the maximum conductivity.

$$\sigma_{\max} = -\frac{m+1}{2\eta} d \ln(R) \tag{14}$$

where η is the intrinsic impedance of the medium, and R is the theoretical reflection coefficient at a normal incidence of the impinging plane wave and m is the profile order and α is not scaled and assumed to be constant.

III. Reflection Error

The example computes the fields generated by a current line which radiates TM polarized waves in free space. To validate the proposed implementation, we evaluate the existence of spurious field due to reflection from the CFS-PML boundaries.

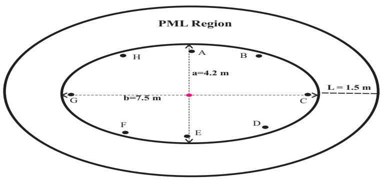

Figure 2: Points on the geometry of which reflection error were calculated.

The computational geometry showed in Fig.2, is a ellipse with axis $a = 4.2$m and $b = 7.5$m. The PML parameters are computed using the above expressions for $m = 2.32765$, $k_{max} = 1$, $R = \exp(-8.72447)$, and $\alpha = 1.9922 \times 10^{-3}$. The physical depth of the PML region is $L = 1.5$m and it is terminated by a perfectly electric wall. The domain was discretized with a mesh with 160,289 triangles and the time step is 0.01583 ns.

To evaluate the performance of the CFS-PML in our implementation, we measure reflection error (in dB) relative to a reference solution [3]. Fig.2 shows points on the geometry where the reflection error was calculated. The excitation is a Ricker pulse (15), located at $(x, y) = (0, 0)$ with a center frequency of $f = 300$ MHz given by

$$G(t) = -2\sqrt{\frac{\exp(1)}{A}} \exp\left(-A(t-B)^2\right) A(t-B) \tag{15}$$

where $A = 2\pi^2 f^2$ and $B = 1/f$.

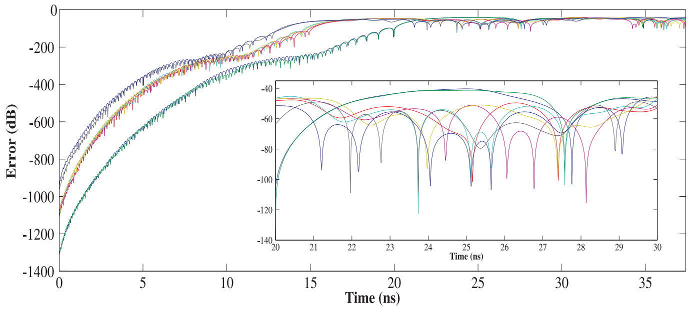

Figure 3: Error of reflection in points A, B, C, D, E, F, G, and H of geomertry

Fig. 3 shows the error of reflection with respect to time in the points of geometry. It is observed that the error is below -40db at all points. This example shows that the proposed CFS-PML for curved geometry has great performance for the reflections caused by the absorption region.

IV. Conclusion

In this paper, we presented a simple algorithm that solve the time dependent Maxwell's equations in curved domains truncated by the CFS-PML. In order to validate our implementation, we evaluate the presence of spurious field due to reflection from the CFS-PML boundaries. The reflection error is consistently below -40dB, which shows that the CFS-PML layer can effectively absorb outgoing waves.

Acknowledgment

This work was supported in part by the State of Minas Gerais Research Foundation - FAPEMIG, Brazil, State of Espirito Santo Research Foundation - FAPES, Brazil, the Brazilian government agency CAPES and National Council for Scientific and Technological Development - CNPq, Brazil.

References

[1] J. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, vol. 114, pp. 185 –200, 1994.

[2] M. Kuzuoglu and R. Mittra, "Mesh truncation by perfectly matched anisotropic absorbers in the finite element method," *Microwave Opt Technol Lett*, vol. 12, pp. 136 –140, 1996.

[3] B. Donderici and F. Teixeira, "Mixed finite-element time-domain method for transient maxwell equations in doubly dispersive media," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 56, no. 1, pp. 113 –120, jan. 2008.

[4] A. S. Moura, R. R. Saldanha, E. J. Silva, M. F. Pantoja, A. C. Lisboa, and W. G. Facco, "Discretization of the cfs-pml for computational electromagnetics using discrete differential forms," *Microwave and Optical Technology Letters*, vol. 55, no. 2, pp. 351–357, 2013. [Online]. Available: http://dx.doi.org/10.1002/mop.27298

[5] A. S. Moura, R. R. Saldanha, E. J. Silva, A. C. Lisboa, W. G. Facco, and N. Z. Lima, "A recursive sparsification of the inverse hodge matrix," *Magnetics, IEEE Transactions on*, vol. 48, no. 2, pp. 611 –614, feb. 2012.