Near-infrared Invisibility Cloak Engineered with Two-phase Metal-dielectric Composites

Koppány Körmöczi and Zsolt Szabó
Department of Broadband Infocommunications and Electromagnetic Theory, Budapest University of Technology and Economics
Egry József u. 18, H-1111 Budapest, Hungary
kormoczik@freemail.hu, szabo@evt.bme.hu

Abstract—**Electromagnetic cloaking device constructed with composite materials, which operates in the range of near-infrared light is presented. The invisibility cloak is designed with transformation optics and consists of fifteen concentric rings. Each ring is fabricated from different anisotropic two-phase metal-dielectric composite. The anisotropic material parameters of the nano-composites are expressed with the Maxwell-Garnett mixing rule. The finite element simulation of the concealment produced by the multilayer nanocomposite is discussed. The geometry of the inclusions and the material parameters of the composites are determined with optimization.**

***Index Terms*—Metamaterials, Nanocomposites, Finite element method, Optimization.**

I. INTRODUCTION

Artificial structures with the goal to fulfil a required electromagnetic behaviour have been engineered for several decades. Nowadays the metamaterial research extends in the direction to engineer arbitrary electromagnetic material properties. Transformation optics is a new approach to design electromagnetic structures [1]-[2], where the paths of electromagnetic waves are controlled with a prescribed spatial variation in the effective material parameters. The propagation of the waves is controlled through a coordinate transformation between the real-space and a virtual-space, in which the wave trajectories are determined by the metrics of this non-real space. Applying this method several electromagnetic devices, including cloaks can be designed [3]-[4], however extremely complex material parameters are required. Cloaking of scattering object was demonstrated first at microwave frequencies, later at terahertz and even at optical frequencies. Several cloaks have been proposed with metamaterials built of split ring resonators [3] or dielectric photonic crystals [4].

In this paper near-infrared cylindrical cloak made of two-phase nanocomposite multilayers is presented and the details of the design algorithm are discussed. The Maxwell equations, in the presence of the anisotropic cloak, are solved with the Matlab's finite element toolbox. Optimization is performed to determine the parameters of the composite. The efficiency of the concealment is demonstrated with the developed full wave solver.

II. NUMERICAL SIMULATION OF THE ELECTROMAGNETIC CONCEALMENT

To simulate the electromagnetic cloaking the Matlab Partial Differential Equation (PDE) Toolbox, a general frequency domain finite element solver is employed to solve the anisotropic, inhomogeneous wave equation derived from the Maxwell's equations. The scattering object is a copper cylinder standing in air with infinite length along the z axis surrounded with the cloak which is wavelength thick. Assuming TM illumination the electromagnetic problem can be reduced to two-dimensions.

The exact material parameters of the cloak can be calculated with the transformation optics and in [3] they are provided in cylindrical coordinates. However the numerical PDE solver of Matlab operates with Cartesian-based directional derivatives. The Cartesian-based relative electric permittivity tensor is obtained with matrix similarity transformation. In the Matlab PDE template of the wave equation the inverse of the permittivity tensor is required. If the cylindrical-based electric permittivity tensor is $\varepsilon_{r,Cyl} = diag[\varepsilon_{rr}(r), \varepsilon_{\Theta\Theta}(r)]$ then the components of the inverted matrix in Cartesian coordinates are

$$\varepsilon_{r,Car}^{-1} = \begin{bmatrix} \frac{\sin^2\Theta}{\varepsilon_{\Theta\Theta}(r)} + \frac{\cos^2\Theta}{\varepsilon_{rr}(r)} & \frac{\cos\Theta\,\sin\Theta}{\varepsilon_{rr}(r)} - \frac{\cos\Theta\,\sin\Theta}{\varepsilon_{\Theta\Theta}(r)} \\ \frac{\cos\Theta\,\sin\Theta}{\varepsilon_{rr}(r)} - \frac{\cos\Theta\,\sin\Theta}{\varepsilon_{\Theta\Theta}(r)} & \frac{\cos^2\Theta}{\varepsilon_{\Theta\Theta}(r)} + \frac{\sin^2\Theta}{\varepsilon_{rr}(r)} \end{bmatrix}, \qquad (1)$$

where the distance $r = \sqrt{x^2 + y^2}$ is measured from the centre of the cylindrical scatterer, the angle $\Theta = \tan^{-1}(y/x)$ and $\varepsilon_{rr}(r)$ and $\varepsilon_{\Theta\Theta}(r)$ are the cylindrical components of the relative electric permittivity, which are functions of r due to the rotational symmetry. The required relative magnetic permeability $\mu_r = \mu_{zz}(r)$ reduces to a scalar for TM mode.

Comparing the explicit PDE form of the inhomogeneous wave equation and the template of the elliptic equation in Matlab PDE Toolbox [5], the correspondences are

$$\begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} = \begin{bmatrix} \varepsilon_{yy}^{-1} & -\varepsilon_{yx}^{-1} \\ -\varepsilon_{xy}^{-1} & \varepsilon_{xx}^{-1} \end{bmatrix},$$
$$a = -\frac{\omega^2}{c^2}\,\mu_r = -k^2\mu_{zz}(r),$$
$$f = 0, \qquad (2)$$

where ω is the angular frequency, k is the wavenumber and c is the speed of light in vacuum. The scalar function u of the template corresponds to the magnetic field intensity H_z.

III. CONCEALMENT WITH MULTILAYER NANOCOMPOSITES

The geometry and the scattering object are similar to those of [3], however herein the operational frequency is 300 THz. At infrared frequencies it is not feasible to cloaking with metamaterials built of split ring resonators. Optimal efficiency of the concealment can be obtained when the cloak is ring with the thickness of one wavelength. The inner and the outer radii of the cloak are 1 µm and 2 µm respectively, therefore it would be very challenging from fabrication point of view to use split ring resonators. Therefore the cloak is divided into fifteen concentric ring layers, each layer having the same thickness, but built from different anisotropic two-phase nanocomposite.

To decrease the difficulties of implementing the anisotropic material parameters of the cloak, the microwave experiment [3] is built with reduced material parameters, which has the same dispersion as the exact material parameters. In this paper different set of reduced parameter are derived so that the magnetic materials can be avoided. In cylindrical coordinates the material parameters of the cloak are

$$\mu_r = 1,\ \varepsilon_r = diag\left[\left(\left(\frac{r-r_a}{r}\right)\left(\frac{r_b}{r_b-r_a}\right)\right)^2, \left(\frac{r_b}{r_b-r_a}\right)^2\right], \tag{3}$$

where $r_a = 1\mu m$ is the inner and $r_b = 2\mu m$ is the outer radius of the cloak and the distance $r \in [r_a, r_b]$. For the elements of the anisotropic tensor the following constraint applies

$$\varepsilon_{rr}(r) \in [0,1],\ \varepsilon_{\Theta\Theta}(r) = 4. \tag{4}$$

The layers of the electromagnetic cloak are engineered with two-phase nano-composites, where the matrix is a dielectric, while the inclusions are metallic nanoparticles with spheroid shape. The anisotropic material parameters of the cloak can be obtained with oriented ellipsoid inclusions. The anisotropic Maxwell-Garnett mixing rule [6] provides closed form expression for the effective electric permittivity of the composite

$$\varepsilon_{eff} = \varepsilon_h + \xi \frac{(\varepsilon_i - \varepsilon_h)\varepsilon_h}{\varepsilon_h + (1-\xi)L(\varepsilon_i - \varepsilon_h)}, \tag{5}$$

where ε_i and ε_h are the relative permittivity of the inclusions and host, ξ is the filling factor and L is the Lorentz depolarization factor, which depends only on the shape of the inclusion. For cylindrical cloaks illuminated under TM conditions, the inclusions are infinitely long elliptical cylinders in the *z* axis and the components of L tensor are

$$L_x = \frac{e}{e+1},\ \ L_y = \frac{1}{e+1},\ \ L_z = 0, \tag{6}$$

where r_x, r_y are the semi-axes of the inclusions with ellipse cross sections in the *x-y* plane, and $e = r_y/r_x < 1$. In each layer of the cloak the orientation of the inclusions is rotationally symmetric with the major axis in Θ and the minor axis in the r direction.

The parameters of the composite are optimized with Differential Evolution algorithm [7] for every layer of the cloak. The search algorithm looks for the optimal composite parameters in a five dimensional parameter space defined as

$$\boldsymbol{P}_i = [r_x, r_y, \xi, \varepsilon_i, \varepsilon_h], \tag{7}$$

by minimizing the difference between the reduced material parameters

$$\varepsilon_{rr}^{disc}[i] = \varepsilon_{rr}\left(a + \frac{d}{2} + (i-1)d\right), \tag{8}$$

where $d = 66.\dot{6}$ nm is the thickness of one layer and electric permittivity calculated with the anisotropic Maxwell Garnett formula. The constant values $\mu_{zz}^{disc}[i] = 1$ and $\varepsilon_{\theta\theta}^{disc}[i] = 4$ are assumed. The optimization is performed for each layer separately in order to determine the effective material parameters of the composites.

Simulation of the electromagnetic concealment produced by the optimized multilayer nanocomposite cloak is presented in Fig. 1. The excitation is a line source generating cylindrical waves. The figure shows that the multilayer nanocomposite can hide the scattering object.

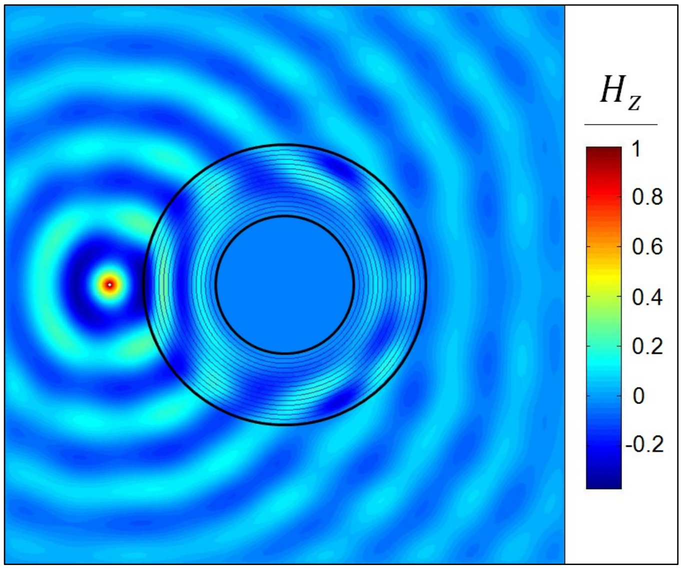

Fig. 1. Simulation of the electromagnetic concealment produced by the multilayer nanocomposite. The excitation is a point source and the *z* component of the magnetic field intensity is shown.

IV. CONCLUSION

We have presented a method, and derived the required mathematical expressions to design near-infrared cylindrical cloaks made of two-phase nanocomposite multilayers. Our software implementing the presented procedure will be published online. The algorithm and the animations presenting the wave propagation around the cloaked object can be useful in advanced electromagnetic courses.

ACKNOWLEDGEMENT

This work has been supported by the János Bolyai Research Fellowship of the Hungarian Academy of Sciences.

REFERENCES

[1] U. Leonhardt, "Optical Conformal Mapping," *Science*, vol. 312, pp. 1777-1780, 2006.
[2] J. B. Pendry, D. Schurig and D. R. Smith, "Controlling Electromagnetic Fields," *Science*, vol. 312, pp. 1780-1782, 2006.
[3] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr and D. R. Smith, "Metamaterial Electromagnetic Cloak at Microwave Frequencies," *Science*, vol. 314, pp. 977-980, 2006.
[4] J. Valentine, J. Li, T. Zentgraf, G. Bartal and X. Zhang, "An Optical Cloak Made of Dielectrics," *Nature Materials*, vol. 8, pp. 568-571, 2009.
[5] K. Körmöczi and Zs. Szabó, "Simulation of transformation optics designed metamaterial for electromagnetic cloaking," *Periodica Polytechnica*, Budapest, to be published.
[6] A. Sihvola, *Electromagnetic Mixing Formulas and Applications*, Institution of Electrical Engineers, London, 1999.
[7] K. V. Price, R. M. Storn and J. A. Lampinen, *Differential Evolution, A Practical Approach to Global Optimization*, Springer, 2005.