# Calculation of the Lightning Electromagnetic Fields Using Non-uniform FDTD Mesh with Soil Ionization

Taobin Jin, Boyuan Zhang, Yuelong Jia, Jun Zou, and Jiansheng Yuan Department of Electrical Engineering, Tsinghua University 100084, Beijing, China (Taobin Jin: Tel.: +86 10 6279 1807, jintaobin@163.com)

Abstract—This paper deals with the finite-difference timedomain (FDTD) calculation of the lightning electromagnetic fields in the air and inside a finitely conducting ground with non-uniform mesh considering soil ionization. Non-uniform mesh in the FDTD is adopted to make the FDTD calculation more efficient, and a comparison between the results of nonuniform mesh and the results of uniform mesh is presented to prove the correctness of the proposed approach. Three different values for the ground conductivity and a soil ionization model are considered in the calculations. The results obtained using FDTD and those in other papers are identical. It is also shown that the approach proposed is able to calculate the electromagnetic fields with the soil ionization considered.

Index Terms—FDTD, lightning, non-uniform mesh, soil ionization.

#### I. INTRODUCTION

A great deal of attention has been devoted recently to the problem of evaluating the lightning electromagnetic fields in the air and inside a finitely conducting ground [1-3]. The finite-difference time-domain (FDTD) method can be utilized to analyze the lightning electromagnetic fields [4-7]. If a surge high current flows in a ground, the soil in the vicinity of the ground electrode would be ionized [7]. In this paper, the attention is paid on how to calculate more efficiently the lightning electromagnetic fields in the air and the underground lightning electromagnetic fields considering soil ionization, using FDTD with non-uniform mesh.

#### II. FDTD METHOD USING NON-UNIFORM MESH

As mentioned earlier, the two-dimensional cylindrical coordinates FDTD using non-uniform mesh is developed to improve the efficiency of the FDTD calculation and help to focus a large number of cells in regions of interest. Simulation requires construction of two mesh regions: a coarse mesh region and a fine mesh region surrounding the area of interest as shown in Fig.1. Once the displacement terms for the fine nodes are obtained, the results from the fine region are used to update the coarse region at the interface of the coarse region and fine region through the relevant interpolation [4]. The average values for the electromagnetic parameters, for example, the dielectric constant and the electrical conductivity have been done to implement the interface condition at the interface of the air region and ground region.

In this paper, we use the subscript "c" to indicate the coarse mesh region variables and the subscript "f" to indicate the fine mesh region variables. Once the displacement terms for the fine nodes are obtained, the results from the fine region are used to update the coarse region at the interface of the coarse region and fine region through simple average values. The first-order Mur absorbing boundary conditions are adopted. In the source region, according to Ampere's law, the difference equation of  $E_{\tau}$  can be treated especially [2].



#### III. SOIL IONIZATION MODEL

If a surge high current flows in a ground, the soil in the vicinity of the ground electrode would be ionized. A soil ionization model, on the basis of the dynamic soil resistivity model, has been adopted, and the resistivity of each soil-representing cell is controlled by the instantaneous value of the electric field there and time in the model [7].

### IV. NUMERICAL RESULTS

The adopted model for the return stroke is the modified transmission line model with exponential decay (MTLE) [1] with a decay constant of 2 km. The return stroke speed is  $1.5 \times 10^8$  m/s. The Heidler current expression and the detailed parameters are given in [1]. The parameters of the soil ionization model  $r_0 = 63 \ \Omega \cdot m$ ,  $E_c = 50 \ \text{kV/m}$ ,  $t_1 = 2 \ \mu\text{s}$ , and  $t_2 = 1 \ \mu\text{s}$  were employed in the FDTD calculations. Two

other different values for the ground conductivity, namely s = 0.01 S/m and s = 0.001 S/m are considered. The relative permittivity is  $e_r = 5$ . The cell sizes are  $\Delta z_c = 3.5$  m,  $\Delta r_c = 3.5$  m,  $\Delta z_f = 1.0$  m,  $\Delta r_f = 1.0$  m, respectively.

## A. FDTD Using Uniform Mesh and Non-uniform Mesh without Soil Ionization

In this example, the underground fields and the fields on the ground are analyzed without soil ionization. A comparison between three different results has been presented in Fig.2 and Fig.3. Dimensions of the uniform fine mesh regions are  $1800 \times 1500$  cells, the calculation consumes 15.1minutes. Dimensions of the non-uniform mesh regions are:  $400 \times 400$  cells for the coarse mesh, the sum of  $400 \times 1500$  cells and  $1400 \times 100$  cells for the fine mesh. The corresponding calculation consumes 4.0 minutes. It can be seen that the results are in excellent agreement with each other and the FDTD with non-uniform mesh is very efficient and valid.



Fig.2. Comparison of the horizontal electric field when r=50m, z=10 m ( $\sigma$  =0.01 S/m and  $\sigma$  =0.001 S/m).



Fig.3. Comparison of the underground horizontal electric field when r=50 m, z=-5 m (  $\sigma$  =0.01 S/m and  $\sigma$  =0.001 S/m).

#### B. FDTD Using Non-uniform Mesh with Soil Ionization

In this example, the underground fields and the fields on the ground are calculated with soil ionization. The soil ionization parameters are mentioned earlier. The results of FDTD have been presented in Fig.4 and Fig.5 with soil ionization and without soil ionization. It can be seen that the results on the ground reach an excellent agreement with each other. The underground fields are identical with the ones without soil ionization, when the distance from the lightning channel is longer than 5 m. Only at the very close to the lightning channel, the underground fields are oscillated for the early time response, but reach an excellent consistency at the last time.



Fig.5. Comparison of the underground horizontal electric field when r=1 m, z=-1 m and r=5 m, z=-2 m.

#### ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant. 51177087.

#### REFERENCES

- Federico Delfino, Renato Procopio, Mansueto Rossi, Farhad Rachidi, and Carlo Alberto Nucci, "Lightning return stroke current radiation in presence of a conducting ground: 2. Validity assessment of simplified approaches", Journal of Geophysical Research, vol. 113, D05111, doi:10.1029/2007JD008567, 2008.
- [2] Abdenbi Mimouni, Federico Delfino, Renato Procopio, and Farhad Rachidi, "On the computation of underground electromagnetic fields generated by lightning: A comparison between different approaches", IEEE Conf. on Power Tech., Lausanne, Switzerland, Jul. 2007.
- [3] Jun Zou, TaoBin Jin, WenWen Li, Jaebok Lee, and Sughun Chang, "Fast numerical evaluation of the horizontal electric field radiated by a lightning channel using the moment technique", IEEE Trans. on Electromagn. Compat., vol. 54, no. 6, pp. 1244-1251, Dec. 2012.
- [4] Tadao Ohtani, Kenji Taguchi, Tatsuya Kashiwa, and Yasushi Kanai, "Overlap algorithm for the nonstandard FDTD method using nonuniform mesh", IEEE Trans. on Magn., vol. 43, no. 4, pp. 1317-1320, Apr. 2007.
- [5] Mikel J White, Magdy F. Iskander, and Zhenlong Huang, "Development of a multigrid FDTD code for three-dimensional applications", IEEE Trans. Antennas Propagat., vol. 45, no. 10, pp. 1512-1517, Oct. 1997.
  [6] Tadao Ohtani, and Yasushi Kanai, "Characteristics of the boundary model
- [6] Tadao Ohtani, and Yasushi Kanai, "Characteristics of the boundary model in the 2-D NS-FDTD method", IEEE Trans. on Magn., vol. 48, no. 2, pp. 191-194, Feb. 2012.
- [7] Ken Otani, Yuki Shiraki, Yoshihiro Baba, Naoto Nagaoka, Akihiro Ametani, and Naoki Itamoto, "FDTD simulation of grounding electrodes considering soil ionization", 2012 International Conf. on Lightning Protection, Vienna, Austria, Sep. 2012.