# A New Quasi-3D Analytical Model of Axial Flux Permanent Magnets Machines

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*Abstract*—Fast design procedures of electrical machines require the use of adapted modeling approaches. This is particularly true for axial flux permanent magnet machines (AFPM), which have an intrinsic three-dimensional (3D) electromagnetic structure. Indeed, accurate modeling of AFPM machines requires the use of 3D finite-element method (3D FEA), which is too time consuming especially at firsts design stages. This paper presents a new quasi-3D analytical model which allows modeling accurately AFPM machines while saving a huge time as compared to 3D FEA. This new approach combines a multislice analytical model, which takes into account a part of the 3D effects, and considers the end effects by a simple and effective radial dependence modeling of the magnetic field in AFPM machines.

Index Terms—Axial flux machines, permanent magnet, electromagnetic modeling, end-effects.

## I. INTRODUCTION

Accurate modelling of AFPM machines requires the use of 3D finite-element method [1]. However, 3D FEA is too time consuming especially at firsts design stages. This paper presents an extension of a previously developed quasi-3D analytical model [2] by taking into account the end effects. This new approach combines a multi-slice analytical model [2], which takes into account a part of the 3D effects, and considers the end effects by a simple and effective radial dependence modeling of the magnetic field [3] in AFPM machines.

The multi-slice analytical model of AFPM machines considers the machine as composed of several linear machines (Fig. 1). Each slice is modeled using a 2D Cartesian model. This approach allows taking into account different magnet shapes and variation of tooth width in the radial direction. In order to take into account the end-effects, the radial dependence of the axial component of the magnetic field is modeled by a function that is able to give precise values in all cases [3].

Results from this new analytical model are compared to those obtained from a multi-slice analytical model, which does not consider the end-effect, and a 3D FEA. It is shown that this new analytical model is more accurate than previous one, and that it constitutes an appreciable alternative to the finite elements method.

### II. QUASI-3D ANALYTICAL MODEL

Figure 1 presents the principle of the quasi-3D model. The machine is divided into a certain number of annular slices in the radial direction. The analytical model based on the solution

of Maxwell equations is established at the average radius ( $R_{mi}$  for the i<sup>th</sup> slice) of each slice (Fig. 1(b)). The analytical solution for the magnetic field distribution is established based on following assumptions: 1) the stator and rotor cores are assumed to be infinitely permeable; 2) eddy current effects are neglected (no eddy current loss in the magnets or armature windings); 3) the permeability of permanent magnets is assumed to be equal to that of air; and finally, 4) the end effects are neglected. Details about the developed analytical model can be found in [2].

End-effects, at inner and outer radii, are take into account by considering the radial dependence of the magnetic field. This radial dependence is modeled by a function that is able to give precise values in all cases [3]:

$$g(r) = \left(\frac{1}{\pi}\right) \cdot \left[ \operatorname{atan}\left(\frac{(r-R_i)}{a}\right) - \operatorname{atan}\left(\frac{(r-R_e)}{a}\right) \right]$$
(1)

where  $R_i$  and  $R_e$  are the inner and outer radii and  $a = \alpha (R_e - R_i)/\tan(\beta \pi/2)$ .  $\alpha$  is used to define the effective radii and  $\beta$  is used to adjust the drop of the axial flux density near the inner and outer radii of the machine.

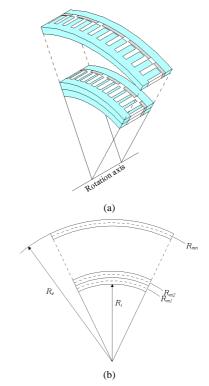


Fig. 1. Subdivision of the axial flux machine in annular slices.

The values of the coefficients  $\alpha$  and  $\beta$  are directly issued from parametric FEM simulations.

The calculation of global quantities (voltages and torques) for the machine is done by considering the contribution of each slice. The total flux linkage and torque are respectively given by

$$\begin{cases} \Phi_t = \sum_{i=1}^{N_c} [g(R_{mi}) \cdot \Phi_i]; \\ T_t = \sum_{i=1}^{N_c} [g^2(R_{mi}) \cdot T_i]. \end{cases}$$

$$(2)$$

where  $N_c$  is the number of considered slices.

# III. VALIDATION OF PROPOSED MODEL

The validation of proposed model is done by comparing open-circuit flux linkage and cogging torque waveforms obtained from the analytical technique and 3D FEA, applied to AFPM machines where the end-effects are particularly significant. Table 1 gives design parameters of axial flux machine to which both the 3D FEA and the quasi-3D analytical models have been applied.  $l_{\alpha}$  e,  $h_s$  and  $w_s$  represent respectively the PM thickness, air-gap thickness, slot height and slot opening. Figure 2 shows a front view of a permanent magnet of studied AFPM machines. The finite element computation are done considering a relative permeability of 1e5 for stator and rotor cores.

Fig. 3 shows comparison of cogging torque waveforms obtained from 3D FEA and analytical models which neglects end-effects (AM without end-effects) and consider them (AM with end-effects). Fig. 4 shows comparison of flux linkage waveforms obtained from both 3D FEA and analytical models, for same machine. It can be noticed that even if results from the different methods agree fairly well, the agreement, of analytical model, with the results issued from the 3D FEA, is improved when end-effects are considered.

TABLE I – MACHINES PARAMETERS

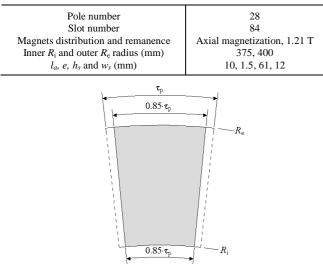


Fig. 2. Magnet shape and outlines of a rotor pole.

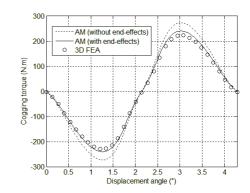


Fig. 3. Cogging torque waveforms comparison.

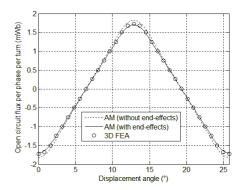


Fig. 4. Open-circuit flux linkage waveforms comparison.

## IV. CONCLUSION

An efficient analytical tool for the design and analysis of axial flux PM machines has been developed. The main features of this analytical design tool have been presented. Compared to 3D finite element computations the developed model gives sufficiently accurate results, while saving a huge computation time.

The analytical models can become an appreciable alternative to the finite elements method especially for 3D problems. This kind of models can be used for parametric design and optimization studies. The presented analytical modeling helps, in a first step, to explore rapidly the search space of potentially optimal prototypes. Obviously, the chosen potentially optimal prototypes issued from this first step have to be refined using a finite element based optimization procedure which acts near the global optimum and then save a large amount of time.

The full paper will contain more details about the proposed approach and present more results to support it.

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