A Differential Permeability 3D Formulation for Anisotropic Vector Hysteresis Analysis

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***Abstract*— In a previous work, we introduced a formulation joining two concepts: the Source-Field technique and the Differential Permeability method [1]. It was shown that the resulting procedure produces accurate results for non-linear cases and it was favorably compared with the more classical approach considering the actual permeability. Here we extend this formulation for vector hysteresis cases taking into account the magnetic anisotropy of the ferromagnetic sheets, which is the main contribution of this paper.**

***Index Terms*—Magnetic anisotropy, magnetic hysteresis, Finite Element Methods.**

I. Introduction

Modeling electrical devices possessing ferromagnetic sheets under variable magnetic fields may require, in certain cases, a high degree of accuracy. If the magnetic circuit does not have airgaps (as transformer yokes), hysteresis plays a relevant role on the device magnetic behavior. The problem becomes more complex if one wishes to consider the electrical sheets anisotropy and the electrical circuit coupling.

In [1] we introduced a formulation based on differential permeability coupled to the source-field method for solving non-linear problems [2]. We pointed out that such an approach would allow solving hysteretic cases, which is now considered in this paper. The hysteresis is modeled by the Jiles-Atherton approach [3][4][5]. Assembling all these concepts in a 3D FE program is the main goal of this paper.

II. The formulation

With the differential permeability method, we have, for isotropic non linear materials

$$\Delta\mathbf{B} = \mu_d \Delta\mathbf{H} \qquad (1)$$

where μ_d is the differential permeability, i.e., the tangent of a typical *B(H)* curve of a non-linear material on an operation point *(B,H)*. **H** and **B** are magnetic field and induction respectively. As the time evolutes from *t* to *t+1*, the variations of **H** and **B** are

$$\Delta\mathbf{B} = \mathbf{B}^{t+1} - \mathbf{B}^{t} \quad \text{and} \quad \Delta\mathbf{H} = \mathbf{H}^{t+1} - \mathbf{H}^{t}$$

With the source-field method [1][2], we have

$$\mathbf{H}^{t+1} = \mathbf{H}_s^{t+1} - grad\,\Omega^{t+1} \qquad (2)$$

where Ω is the magnetic scalar potential and $\mathbf{H}_s$ is the source field associated to the imposed current density as $rot\,\mathbf{H}_s = \mathbf{J}$ [1][2]. In this work, considering magnetic anisotropy [6], (1) is extended to

$$\Delta\mathbf{B} = \|\mu_d\|\Delta\mathbf{H} \qquad (3)$$

where the magnetic differential permeability is defined by

$$\|\mu_d\| = \begin{bmatrix} \mu_{dxx} & \mu_{dxy} & \mu_{dxz} \\ \mu_{dyx} & \mu_{dyy} & \mu_{dyz} \\ \mu_{dzx} & \mu_{dzy} & \mu_{dzz} \end{bmatrix}$$

Considering $\Delta\mathbf{B} = \mathbf{B}^{t+1} - \mathbf{B}^{t}$ and (2), (1) becomes

$$\mathbf{B}^{t+1} = \mathbf{B}^{t} + \|\mu_d\|(\mathbf{H}_s^{t+1} - \mathbf{H}_s^{t} + grad\,\Omega^{t} - grad\,\Omega^{t+1}) \qquad (4)$$

where $\mathbf{B}^t$, $\mathbf{H}_s^t$ and Ω^t are known from the previous time step. The main equation on this development is

$$div\,\mathbf{B}^{t+1} = 0 \qquad (5)$$

or, from (4)

$$div[\mathbf{B}^{t} + \|\mu_d\|(\mathbf{H}_s^{t+1} - \mathbf{H}_s^{t} + grad\,\Omega^{t} - grad\,\Omega^{t+1})] = 0 \qquad (6)$$

Applying the Galerkin method to (5), we have

$$\int_V N\,div\,\mathbf{B}^{t+1}dv = 0 \qquad (7)$$

where N is the nodal shape function of a tetrahedron element and *V* is the domain volume. It is

$$\int_V N\,div\,\mathbf{B}^{t+1}dv = \oint_{s(v)} N\;\mathbf{B}^{t+1}\cdot d\mathbf{s} - \int_V grad\;N\cdot\mathbf{B}^{t+1}dv = 0$$

The first term on the right hand side is related to the classical boundary conditions for scalar potential [1]. The second one must be evaluated for the numerical implementation, as:

$$-\int_V grad\;N\cdot\mathbf{B}^{t+1}dv = 0$$

or, using (4)

$$-\int_V grad\;N\cdot[\mathbf{B}^{t} + \|\mu_d\|(\mathbf{H}_s^{t+1} - \mathbf{H}_s^{t}) + \mu_d(grad\,\Omega^{t} - grad\,\Omega^{t+1})]dv = 0$$

In order to couple the above equation with electrical circuit we introduce the vector **K** as [1]:

$$\mathbf{H}_s = \mathbf{K}\,I_0 \qquad (8)$$

where I_0 is the current flowing in a conductor. We also use the vector **N** expressed by:

$$rot\,\mathbf{H}_s = \mathbf{N}\,I_0 \qquad (9)$$

Considering (8) and (9) above, we have:

$$rot\,\mathbf{K} = \mathbf{N} \qquad (10)$$

Therefore, **K** corresponds to the magnetic field created by a unitary current in a coil wire. Now let us consider the electric circuit coupled to the magnetic structure.

$$V_m = R_m i_m + \frac{d\Phi_m}{dt} \qquad (11)$$

where V_m, R_m, i_m and Φ_m are, respectively, the voltage, the resistance, the established current and the magnetic flux linkage in a generic electric circuit m (suppose that there are M electric circuits). From [1] it can be shown that the magnetic flux is:

$$\Phi^{t+1} = \int_{V_b} \mathbf{B}^{t+1} \cdot \mathbf{K}\, dv \qquad (12)$$

where V_b is the volume of a coil element of an electric circuit m. For the time step $t+1$ we have:

$$V_m^{t+1} = R_m i_m^{t+1} + \frac{d}{dt}\int_{V_b} \mathbf{B}^{t+1} \cdot \mathbf{K}\ dv \qquad (13)$$

Here we apply the derivative time discretization as:

$$\Delta t\, V_m^{t+1} = \Delta t\, R_m i_m^{t+1} + \int_{V_b} (\mathbf{B}^{t+1} - \mathbf{B}^{t}) \cdot \mathbf{K}\ dv \qquad (14)$$

Applying (4) for $\mathbf{B}^{t+1}$ we have all the necessary expressions to obtain the matrix system coupling (7) and (14). It gives [1]:

$$\begin{bmatrix} G^T \|\mu_d\| G & -G^T \|\mu_d\| K \\ -K^T \|\mu_d\| G & \Delta t\, R_m + (K^T \|\mu_d\| K) \end{bmatrix} \begin{bmatrix} \Omega^{t+1} \\ i_m^{t+1} \end{bmatrix} =$$

$$\begin{bmatrix} G^T \|\mu_d\| G & -G^T \|\mu_d\| K \\ -K^T \|\mu_d\| G & (K^T \|\mu_d\| K) \end{bmatrix} \begin{bmatrix} \Omega^{t} \\ i_m^{t} \end{bmatrix} + \begin{bmatrix} G^t B^t \\ \Delta t\, V_m^{t+1} \end{bmatrix}$$

where G is the *grad N* and K is the matrix form of $\mathbf{K}$.

III. EXAMPLE

As example of the formulation, let us we consider the simple structure of Fig. 1. This magnetic circuit has two isotropic pieces P1 and P2 ($\mu = 1000\mu_0$). The first graphical result is shown in Fig. 2. The *Ozx* projection of same result is presented in Fig. 3.

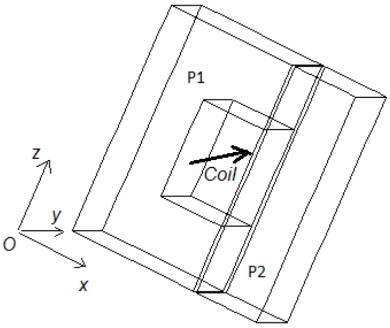

Fig. 1- Magnetic Circuit; part P2 is isotropic

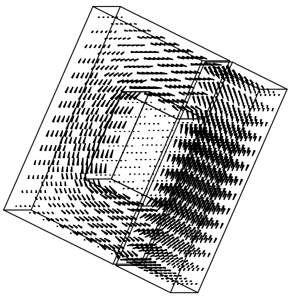

Fig. 2- The whole structure is isotropic

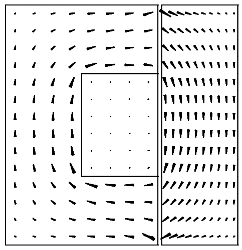

Fig. 3- *Ozx* Projection; P2 is isotropic

In the following figures, we consider that part P2 is anisotropic. Figure 4 is related to results when the permeability tensor of P2 is given as

$$\begin{bmatrix} 1000\mu_0 & 0 & 0 \\ 0 & 100\mu_0 & 0 \\ 0 & 0 & 100\mu_0 \end{bmatrix}$$

where the $\mu_x > \mu_z$. In Fig. 5 we have the flux distribution when the *Oz* permeability component is predominant.

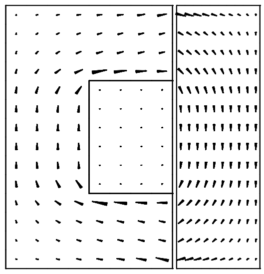

Fig. 4- P2 is anisotropic $(\mu_x > \mu_z)$

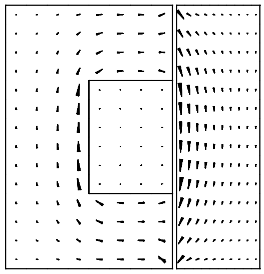

Fig. 5- P2 is anisotropic $(\mu_z > \mu_x)$

IV. CONCLUSION

We presented in this paper an extension of a previous work [1]. Here the anisotropy is included in the formulation and the vector hysteresis phenomenon has been also considered by means of a vectorized Jiles-Atherton model. The latter is under final investigations for convergence improvements. Final results will be shown in the full version of this paper.

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