Influence of Skin Effect on Homogenization of Composite Materials : Application to Shielding Effectiveness

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***Abstract*—Composite materials are more and more used to contribute to structure lightening in electromagnetic shielding applications. The strong interaction involved in composite materials between the microstructure and the electromagnetic field makes this phenomenon difficult to model. Considering details of the microstructures would involve an excessive number of unknowns with standard numerical tools. Homogenization is a possibility to overcome this problem. The equivalent homogeneous medium obtained with such methods can then be introduced into numerical tools to model full shielding enclosures. A recent homogenization model has been presented to obtain the equivalent homogeneous medium of materials submitted to electromagnetic waves through the introduction of a length parameter. This previous work was limited to frequencies with no skin effect involved. This paper extends it to higher frequencies by considering this effect.**

***Index Terms*—Effective medium, Electromagnetic compatibility, Homogenization, Maxwell-Garnett model, skin-effect.**

I. Introduction

Electromagnetic shielding enclosures, which contribute to electromagnetic compatibility, are often made of metal alloys. However, the aerospace industry is interested in the use of composites to reduce the weight of aircraft. This is the case of composite materials made of epoxy resin surrounding carbon inclusions used in many applications nowadays. It is important to be able to model such materials in order to design these materials in a suitable way for the desired application. Numerical tools cannot be used because of the large number of unknowns generated to take into account every details of the microstructures. The use of homogenization tools is a good way to get over this problem. Indeed, many homogenization methods have been developed to model composite materials [1]–[5].

An homogenization model, able to provide the effective properties for higher frequencies than classical models, has been previously presented [6]. It extends the frequency range of classical models and remains accurate until the skin-depth reaches the size of heterogeneities. This work will extend its scope to higher frequencies by considering the skin effect (SKE). It is introduced in the model in a similar way than previous studies [7], [8].

II. Dynamic homogenization model (DHM)

The homogenization model considered in this study is based on the inclusion problems [9], [10]. The properties of the equivalent homogeneous medium (EHM) are obtained from the study of different basic inclusion problems. There are as much inclusion problems than phases in the real material. Each phase is considered to behave on average as an inclusion embedded in an infinite matrix. The shape of this representative inclusion depends on the phase distribution in the real material. It has been shown that particular choices of the homogeneous infinite medium (HIM) lead to classical homogenization models [9].

This model needs, as an input, the phase distribution which is considered trough the depolarizing tensor. A length parameter has been introduced in the definition of the HIM to adapt this model, developed in a quasistatic approach, to higher frequencies. This parameter γ, named the characteristic size of the microstructure, needs to be defined for each different microstructure.

A. Effective permittivity

For biphasic composites, with isotropic constituents, the sum of the two inclusion problems can be written [6]:

$$\widetilde{\epsilon_u^*} = \frac{\epsilon_1^* \dfrac{f_1}{\epsilon_\infty^* + N_u\left(\epsilon_1^* - \epsilon_\infty^*\right)} + \epsilon_2^* \dfrac{f_2}{\epsilon_\infty^* + N_u\left(\epsilon_2^* - \epsilon_\infty^*\right)}}{\dfrac{f_1}{\epsilon_\infty^* + N_u\left(\epsilon_1^* - \epsilon_\infty^*\right)} + \dfrac{f_2}{\epsilon_\infty^* + N_u\left(\epsilon_2^* - \epsilon_\infty^*\right)}} \quad (1)$$

In this expression, ϵ_∞^* is the complex permittivity of the HIM in the inclusion problems and N_u is the depolarization factor in the direction $\mathbf{u}$. This expression gives the macroscopic complex permittivity of the EHM $\widetilde{\epsilon_u^*}$ in terms of ϵ_1^* and ϵ_2^* who are the complex permittivities of the matrix (Phase 1) and the inclusions (Phase 2) respectively. These complex permittivities are written :

$$\epsilon_i^* = \epsilon_i - j\frac{\sigma_i}{\omega} \quad (2)$$

with ϵ the electric permittivity, σ the electric conductivity and ω the angular frequency.

B. Infinite Medium

The infinite medium chosen in [6] which extends the model from quasistatic approach to electromagnetic waves is:

$$\epsilon^*_\infty = \epsilon^*_1 + \epsilon^*_2 \left(\frac{\gamma}{\lambda}\right)^2 \tag{3}$$

where γ is the characteristic length of the microstructure and λ is the wavelength in the EHM. In the case of a square array of conductive discs, the characteristic length of the microstructure is the diameter of the inclusions [6]. At low frequencies (high value for λ), the second part of the infinite medium vanishes and the model become equivalent to Maxwell-Garnett model (MGM) which is known to work quite well for a small range of frequency.

The SKE is taken into account in the infinite medium through the fibers conductivity :

$$\epsilon^*_2 = \epsilon_2 - j\frac{\overline{\sigma}_2}{\omega} \tag{4}$$

where $\overline{\sigma}_2$ is the effective conductivity of the fibers. When the diameter of the fibers is much higher than the skin-depth, it can be noticed that the central part of the fibers do not exhibit induced current. That is to say that the effective surface of fibers is scaled by the field distribution [7], [8]. The definition of this term $\overline{\sigma}_2$ will be more detailed in the full paper.

III. Modeling results

To evaluate the accuracy of the homogenization model presented in the previous section, the shielding effectiveness (SE) of a sheet made of the equivalent homogeneous material is compared to the shielding effectiveness of an heterogeneous sheet computed by finite element method (FEM) on a mesh of the complete microstructure. Fig. 1 presents the studied domain in FEM. The infinity of the sheet is taken into account through periodic conditions.

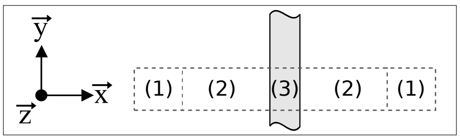

Figure 1. Scheme of Shielding Effectiveness computations configuration including FEM domain: Perfect Matched Layers (PML) (1), Air (2) and studied sheet (3).

As shown on Fig. 2, the sheet made of the EHM computed with the DHM considering the SKE provides a SE similar to FEM modeling. While the accuracy of MGM decreases, the DHM considering the SKE keeps relevant until high frequency.

IV. Conclusion

The dynamic homogenization method presented in this paper provides the equivalent homogeneous medium for biphasic microstructures submitted to high frequency electromagnetic waves. The limitation involved by the skin-depth effect in the previous model has been overcome.

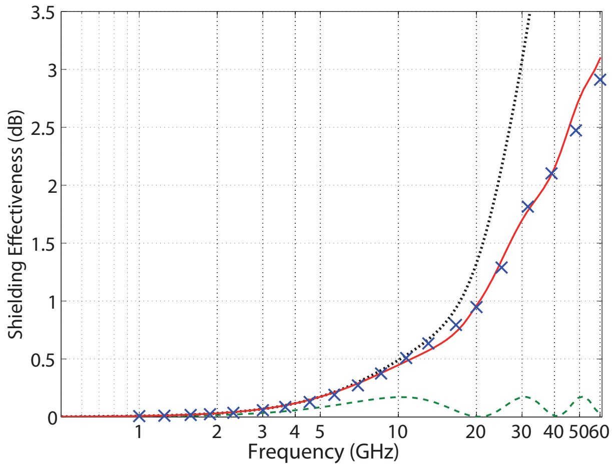

Figure 2. SE of the periodic microstructure computed with FEM (crosses), the DHM (dot-dash line), the DHM considering the SKE (line) and Maxwell Garnett model (dashed line). Constituents properties : $\sigma_1 = 1e - 20\ S.m^{-1}$, $\sigma_2 = 10000\ S.m^{-1}$ and $\epsilon_1 = \epsilon_2 = \epsilon_0$

The results obtained are similar to FEM, but computed in a much shorter time and the frequency domain of validity is extended regarding Maxwell-Garnet homogenization model. The equivalent homogeneous medium obtained can be introduced into numerical tools to model the behavior of complete devices. The full-paper will present the comprehensive approach with more details and modeling results will be compared to a discontinuous Galerkin time domain computation method.

Acknowledgements

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References

[1] A. Sihvola, "Electromagnetic Mixing Formulas and Applications", *IEE Electromagnetic Waves Series 47* (1999).

[2] G.W. Milton, "The Theory of Composites", *Cambridge University Press* (2002).

[3] E. F. Kuester, C. L. Holloway, "Comparison of Approximations for Effective Parameters of Artificial Dielectrics", *IEEE Trans. Microw. Theory Techn.*, **38**(11), (1990), pp. 1752-1755.

[4] M.Y. Koledintseva, J. Drewniak, R. Dubroff, K. Rozanov, B. Archambeault, "Modelling of Shielding Composite Materials and Structures for Microwave Frequencies", *PIERB*, **15**, 197-215 (2009).

[5] M.S. Sarto, A.G. D'Aloia, A. Tamburrano, G. De Bellis, "Synthesis, Modeling, and Experimental Characterization of Graphite Nanoplatelet-Based Composites for EMC Applications", *IEEE Trans. Electromagn. Compat.*, **54(1)**, 17-27 (2012).

[6] V. Préault, R. Corcolle, L. Daniel, L. Pichon, "Effective permittivity of shielding composite materials for microwave frequencies", *IEEE Trans. Electromagn. Compat.*, (under revision).

[7] M. Y. Koledintseva, R. E. DuBroff, R. W. Schwartz, "A Maxwell Garnett Model for Dielectric Mixtures Containing Conducting Particles at Optical Frequencies", *PIER*, **63**, (2006), pp. 223-242.

[8] A. Bossavit, "A High-frequency homogenization technique", *Proceedings of STAMM 2010*, Berlin, Germany, (2010).

[9] M. Bornert, T. Bretheau, P. Gilormini, "Homogénéisation en mécanique des matériaux", *Hermes Science* (2001).

[10] L. Daniel, R. Corcolle, "A Note on the Effective Magnetic Permeability of Polycrystals", *IEEE Transactions On Magnetics*, **43**(7), (2007), pp. 3153-3158.