Vector Generalization of Uniaxial Models for Magnetomechanical Hysteresis and Magnetostriction

Paavo Rasilo[1], Deepak Singh[1], Reijo Kouhia[2], Anouar Belahcen[1], and Antero Arkkio[1]

[1]Aalto University School of Electrical Engineering, Dept. of Electrical Engineering, P.O. Box 13000, FI-00076, Aalto, Finland
[2]Tampere University of Technology, Department of Mechanics and design, P.O. Box 589, 33101, Tampere, Finland

Abstract—We propose a method to extend unidirectional models for stress-dependent magnetic hysteresis and magnetostriction for rotational magnetic fields and biaxial stresses. Analogously to the Mayergoyz vector extension for scalar hysteresis models, the input magnetic flux-density vector and stress tensor are projected into a finite number of directions in a semicircle, the unidirectional model is applied into each direction, and the output magnetic field-strength vector and magnetostriction tensor are formed by summing up the scalar contributions from each direction. The model is found to give physically reasonable results with rotational flux-density excitation with an externally imposed stress. In addition, a visualization method for second-order tensors is proposed.

***Index Terms*—Magnetic hysteresis, magnetoelasticity, magnetostriction.**

I. Introduction

COUPLED magnetomechanical material models are needed for accurate prediction of losses, forces and vibrations in electromagnetic devices. During the past 25 years, models for rotational magnetic hysteresis [1]-[2], uniaxial magnetomechanical hysteretic behavior [3], and biaxial magnetization and magnetostriction in anhysteretic cases [4], [5] have been presented. However, coupling the rotational magnetic hysteresis to biaxial mechanical loading has started to receive more attention only quite recently [6].

In brief, a coupled rotational magnetoelastic hysteresis model should be able to produce the field-strength vector and magnetostriction tensor under arbitrary rotating flux-density excitation and an externally imposed stress tensor. In this paper, we propose a method to generalize unidirectional magnetoleastic models to be used with 2-D fields. The generalization is performed analogously to the Mayergoyz vector extension for rotational hysteresis [1]. We project the input flux density and external stress tensor into different directions in a semicircle, apply the unidirectional model of [3] to these projections and sum up the contributions from each direction. When compared to measurements, the model is found to produce physically reasonable results.

II. Methods

A. Visualization of Tensors

First, in order to visualize the results obtained with the model, we propose a method for graphical presentation of second-order tensors in 2-D. To visualize tensor $\boldsymbol{T}$, it is first used to map a finite set of unit vectors $\boldsymbol{u}_i$ pointing in directions φ_i into a new set of vectors $\boldsymbol{v}_i = \boldsymbol{T}\boldsymbol{u}_i$. Vectors $\boldsymbol{v}_i$ can then be plotted starting from the unit circle at angles φ_i. Fig. 1 shows four examples of tensors visualized by this method. The ratio and signs of the diagonal terms, the principal directions and the rotational nature of the tensors can be seen from the plots. Unlike the Mohr's circle or the approach of [7], this method can also visualize asymmetric tensors.

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B. Unidirectional Model

In this paper, the unidirectional magnetomechanical hysteretic behavior is modeled with the scalar Sablik-Jiles-Atheron (SJA) model [3]. In brief, the model is able to produce the magnetic field strength h and magnetostriction λ as a function of the flux density b and an external stress σ imposed parallel to the flux density. The governing equations of the model are similar to the traditional Jiles-Atherton model, except that the effective field has an additional term h_σ which depends on the stress and the derivative of the magnetostriction with respect to the magnetization m:

$$h_\sigma = \frac{3}{2}\frac{\sigma}{\mu_0}\frac{\partial \lambda}{\partial m}. \tag{1}$$

Instead of using the magnetostriction model proposed in [3] we choose here a simpler experimental model and express the magnetostriction as

$$\lambda(m,\sigma) = \left(a_1 + \tanh\frac{a_2-\sigma}{a_3}\right)\left(a_4 m^2 + a_5 m^4 + a_6 m^6\right), \tag{2}$$

in which the six parameters a_i, i = 1,…,6 are fitted by comparison to measurements with an Epstein frame. The

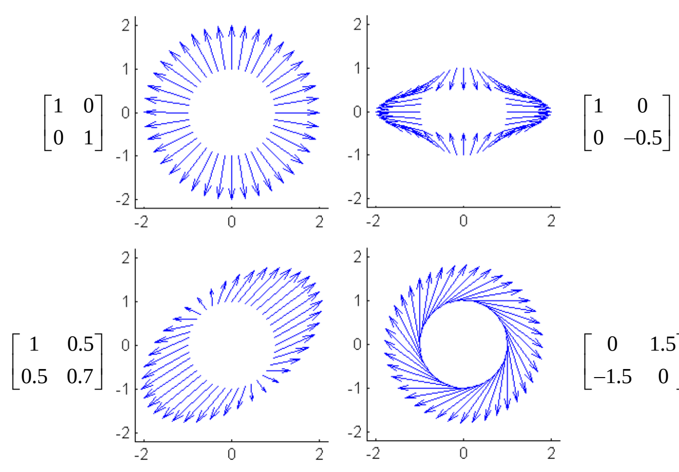

Fig. 1 Visualization examples of certain second-order 2×2 tensors.

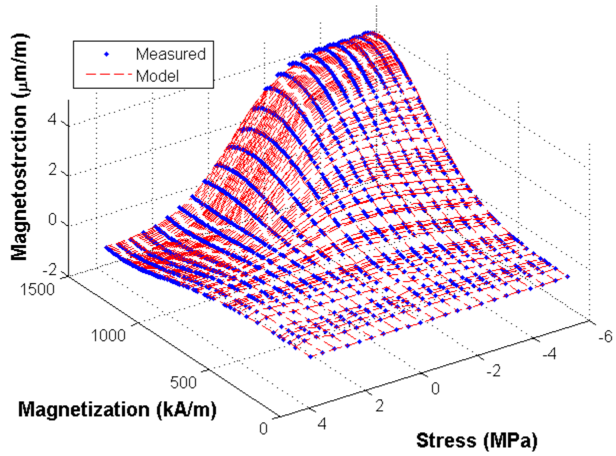

Fig. 2 Measured magnetostriction as a function of stress and magnetization and the best fit using model (2).

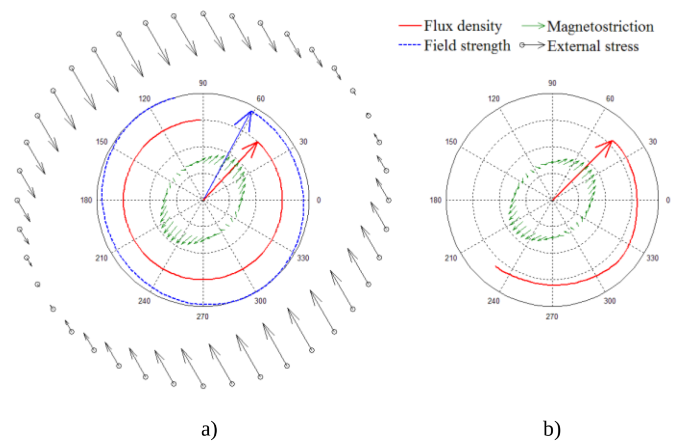

Fig. 3 a) Modeled and b) measured magnetostriction with rotating flux density. The simulated external stress and field strength are also shown in a).

hyperbolic tangent function ensures saturation of the peak magnetostriction with respect to the stress and provides a sufficient fit to measurements, as shown in Fig. 2.

C. Vector Model

The vector generalization of the scalar model is based on the approach of Mayergoyz, who extended the scalar Preisach hysteresis model to vector fields in [1]. In this method, the flux-density vector is projected into a finite number n_φ of directions $\boldsymbol{u}_i$ in a semicircle $\varphi_i \in [-\pi/2, \pi/2]$, the scalar model is applied into these directions, and the output field-strength vector is obtained as a sum of the scalar contributions from each direction. In a similar manner, we project both the flux density $\boldsymbol{b}$ and the stress $\boldsymbol{\sigma}$:

$$b_i = \boldsymbol{b}\cdot\boldsymbol{u}_i \text{ and } \sigma_i = (\boldsymbol{\sigma}\boldsymbol{u}_i)\cdot\boldsymbol{u}_i\,, \tag{3}$$

and input these projections to the unidirectional model. The output field strength and magnetostriction are then obtained in a local coordinate system aligned with $\boldsymbol{u}_i$:

$$\boldsymbol{h}_i(\boldsymbol{b},\boldsymbol{\sigma}) = \begin{bmatrix} h(b_i,\sigma_i) \\ 0 \end{bmatrix} \text{ and } \boldsymbol{\lambda}_i(\boldsymbol{b},\boldsymbol{\sigma}) = \lambda(b_i,\sigma_i)\begin{bmatrix} 1 & 0 \\ 0 & -0.5 \end{bmatrix}. \tag{4}$$

The field strength is parallel to $\boldsymbol{u}_i$ while the magnetostriction has principal components parallel and perpendicular to $\boldsymbol{u}_i$, the latter defined by the magntostrictive Poisson's ratio 0.5.

The vector model output is then formed by transforming $\boldsymbol{h}_i$ and $\boldsymbol{\lambda}_i$ back to a global coordinate system and summing up the contributions from each direction:

$$\boldsymbol{h}(\boldsymbol{b},\boldsymbol{\sigma}) = \sum_{i=1}^{n_\varphi} \boldsymbol{R}_i^{\mathrm{T}}\boldsymbol{h}_i(\boldsymbol{b},\boldsymbol{\sigma}) \text{ and } \boldsymbol{\lambda}(\boldsymbol{b},\boldsymbol{\sigma}) = \sum_{i=1}^{n_\varphi} \boldsymbol{R}_i^{\mathrm{T}}\boldsymbol{\lambda}_i(\boldsymbol{b},\boldsymbol{\sigma})\boldsymbol{R}_i\,, \tag{5}$$

with

$$\boldsymbol{R}_i = \begin{bmatrix} \cos\varphi_i & -\sin\varphi_i \\ \sin\varphi_i & \cos\varphi_i \end{bmatrix} \tag{6}$$

being the transformation matrices between the global and local coordinate systems.

III. Results and Discussion

The model was applied to calculate the magnetostriction tensor and field-strength vector for a circular flux density (1.5 T) and an external 300-MPa compressive stress with its principal axis in the direction of 120°. The unidirectional magnetostriction was obtained from (2) but otherwise the SJA model parameters were equal to those in Fig. 1 of [3]. $n_\varphi = 32$ directions were used in the angular discretization.

The locus of Fig. 3 a) clearly shows that more field strength is needed in the direction of the principal axis of the compressive stress in order to keep the flux density circular. When the magnetostriction is compared to measurements with a single-sheet tester in Fig. 3 b), a similar instantaneous behavior is observed. Although the sample was not stressed during the measurements and comprehensive identification measurements are yet to be done, the model can be concluded to produce physically reasonable results for both the field strength and magnetostriction. In the future, measurements with stressed samples will be done for further comparison.

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