A Jiles-Atherton based hysteresis model for magnetic materials under complex magneto-mechanical loadings

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***Abstract*—Accurate electromagnetic design must take into account the multiaxial magneto-mechanical loadings experienced by electromagnetic devices. It should also include the description of hysteresis behavior which is of primary importance in the efficiency of these devices. Based on a vectorial extension of the classical Jiles-Atherton model, we introduce the magneto-mechanical effects using a multi-scale anhysteretic approach. The model allows to describe with reasonable accuracy the permeability, coercive field and hysteresis losses density of magnetic materials submitted to magneto-mechanical loadings. It is then used in a time-stepping finite element model to evaluate the effect of the stress due to binding process on the hysteresis losses in the rotor of a switched reluctance motor.**

***Index Terms*—Electric machines, Magnetomechanical effects, Magnetostriction, Magnetic materials.**

I. Introduction

In many applications, the resolution of coupled magneto-mechanical problems must be addressed considering the complex hysteretic constitutive law of ferromagnetic materials. Because of the multiaxial nature of the magneto-mechanical loadings, predictive models must be implemented in order to study real electrical engineering problems. An anhysteretic multi-scale model (MSM) based on energy considerations at the grain scale and on homogenization/localization procedures at the polycristal scale has been developed [1] to overcome the restrictions of uniaxial models [2]. Here, we show that the information brought by the MSM can be advantageously used to extend the vectorial Jiles-Atherton (JA) [3] model and describe hysteretic magneto-mechanical behaviors.

II. Magnetic hysteresis under stress

The vector generalization of the JA model proposed by Bergqvist [3] is here used as a phenomenological framework to describe magnetic hysteresis. In this model, the vector variation of the magnetization ($d\vec{M}$) must satisfy:

$$d\vec{M} = (\vec{\chi}.(d\vec{H} + \beta d\vec{M}))^{+}\vec{u}_{\chi} + c\, d\vec{M}_{an} \tag{1}$$

with $\vec{\chi} = 1/k(\vec{M}_{an} - \vec{M}) = 1/k\left\|\vec{M}_{an} - \vec{M}\right\|\vec{u}_{\chi}$. In these equations, $\vec{H}$ is the applied magnetic field, β (usually noted α), c and k are the classical JA scalar parameters (isotropic materials are considered here), and M_{an} represents JA's anhysteretic magnetization. It is assumed that mechanical stress affects the susceptibility and the coercive field of the material so that a dependence to both applied magnetic field and stress tensor ($\boldsymbol{\sigma}$) is introduced in the definition of M_{an} and k.

In order to describe the anhysteretic behavior under complex magneto-mechanical loadings, a full MSM approach [1] can be used. However, its practical implementation into numerical analysis tools would lead to prohibitive computation time. This is why we consider a simplified MSM which was shown to be efficient and suitable for 2D finite element analysis [4]. The material is modeled as a single crystal made of a collection of magnetic domains randomly oriented in the modeling plane. At the scale of the magnetic domain, the local magnetization $\vec{M}_{\alpha}$ and magnetostriction strain $\boldsymbol{\varepsilon}^{\mu}_{\alpha}$ depend only on the orientation of the domain $\vec{\alpha}$ and the saturation values of magnetization (M_s) and strain. The local potential energy W_{α} of the material is then written as the sum of magneto-static and elasto-static contributions, $W_{\alpha} = -\mu_0\vec{H}.\vec{M}_{\alpha} - \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^{\mu}_{\alpha}$. The volume fraction of each domain family f_{α} is then calculated using a Boltzmann type relation:

$$f_{\alpha} = \exp(-A_s.W_{\alpha}) / \int_{\alpha} \exp(-A_s.W_{\alpha}) \tag{2}$$

where A_s is a material parameter linked to the initial anhysteretic susceptibility [4]. The macroscopic anhysteretic magnetization is finally obtained thanks to an averaging operation over all possible directions.

Considering now the JA model, the parameter k directly acts on the coercive field and is strongly related to the density of pinning sites and wall displacements [5]. We consider a twofold correction of the no-load value k_0:

$$k = k_0\left(1 - \kappa_r\frac{M_{an}}{M_s}\right)\left(1 + \kappa_f\left(1 - \frac{\pi}{2}\int_0^{2\pi} f_{\alpha}\left|\vec{\alpha}.\vec{m}\right| d\alpha\right)\right) \tag{3}$$

where κ_r and κ_f are constant parameters, and $\overrightarrow{m} = \overrightarrow{dM}/dM$. The first term accounts for the fact that the contribution of wall bending increases as the magnetization increases. This kind of correction was originally introduced in [5]. The second term aims at accounting for the effect of stress through the volume fractions f_{α} obtained from the MSM. Following the interpretation proposed by Pulnikov [6], the magnitude of wall displacements decreases as the volume fraction of domains "well" oriented with respect to $\overrightarrow{m}$ increases. In the proposed correction, the $(-\pi/2\left|\vec{\alpha}.\vec{m}\right|)$ function could be substituted by

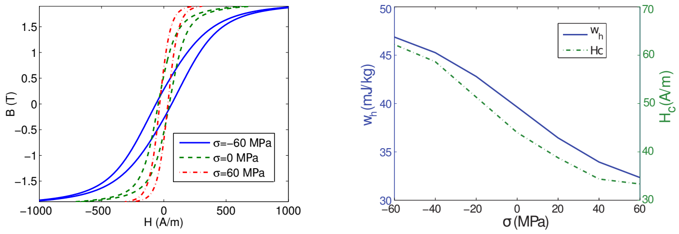

Figure 1: Predicted hysteresis loops (left) and coercive field and hysteresis losses density (right) for an iron-silicon steel under uniaxial stress applied parallel to the magnetic field.

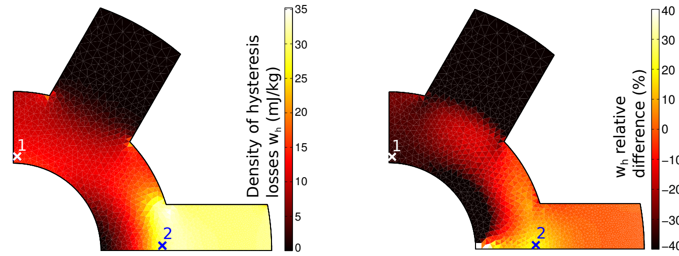

Figure 3: Hysteresis losses density: absolute value (left) and relative value compared to the unstressed configuration (right).

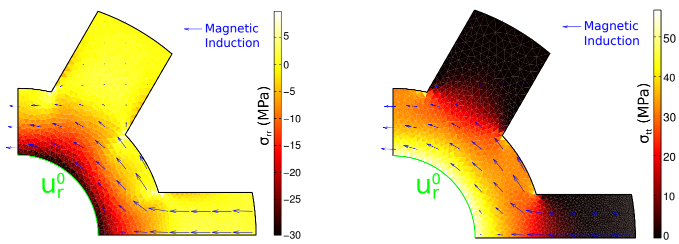

Figure 2: Radial (left) and azimuthal (right) stress distributions. The arrows represent for the magnetic induction.

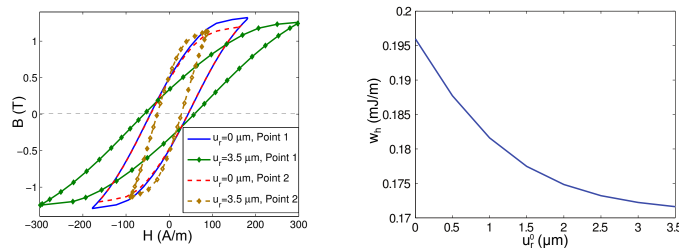

Figure 4: Hysteresis loops at points 1 and 2 and total losses.

other convenient functions of the angle $(\vec{\alpha}, \vec{m})$, decreasing on $[0, \pi/2]$ and symmetric with respect to $k\pi/2$ for any integer k. The $\pi/2$ factor ensures that the correction is zero for a uniform distribution of magnetic domains.

Equation (1) is discretized using centered differences and the expression of the magnetization is kept implicit. The model is tested under uniaxial loading with the set of parameters shown in Table I for Fe-3%Si steel sheets. Parameter κ_r allows to manage the variation of the width of the loop with respect to the magnetization. Parameter κ_f allows to fit the evolution of the coercive field, and hence of hysteresis losses as a function of stress. These parameters are identified from uniaxial tests. Results shown in Fig.1 are consistent with experiments [7].

Table I: Material model parameters

M_s (A.m^{-1})	λ_s	A_s (m^3.J^{-1})	c	β	k_0	κ_r	κ_f
1.6 10^6	10^{-5}	6.6 10^{-3}	0.2	1e-5	66	0.3	0.6

III. Hysteresis losses in the rotor of a SRM

We consider a locked rotor test of a switched reluctance motor (SRM), the mechanical stress is caused by the binding process between the rotor sheets and the shaft. The phase of the stator which is in conjunction is fed by a sine current source. Discarding flux leakage and accounting for the symmetries, only one quarter of the rotor and the air gap are modeled. The stress distribution is computed once using a 2D plane stress linear elasto-static finite element formulation. Magnetostriction strain is neglected. The radial displacement u_r^0 is enforced on the rotor/shaft interface (at $r = 1.15cm$). The magneto-static problem is solved using a 2D scalar magnetic potential formulation. At each time step, the non-linearity is handled using the local coefficient polarization method proposed by Dlala et al. [8]. The magneto-mechanical loading is shown in Fig.2 for $u_r^0 = 3.5\mu m$. The resulting distribution of density of hysteresis losses w_h is computed for different values of the imposed mechanical displacement u_r^0. An example is given on Fig.3 (left). The relative difference between the loss density with and without stress is also presented (right). Finally, the effect of stress on the hysteresis loops at points 1 and 2, and on the total losses are shown in Fig.4.

In conclusion, the proposed model constitutes a new approach to take into account the effects of mechanical stress in magnetic materials. It is computationally efficient for use with FE methods. In a realistic configuration, the complex magneto-mechanical loading is shown to highly affect the hysteresis losses density and then must be carefully considered.

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