# Vector Jiles-Atherton Hysteresis Model and Its Application to Magnetizing Analysis for an Anisotropic Bonded NdFeB Magnet

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*Abstract***—This paper presents a vector Jiles-Atherton hysteresis model for anisotropic bonded NdFeB permanent magnet to estimate the residual magnetic flux density accurately. The finite element analysis procedure taking into account of the vector Jiles-Atherton hysteresis model is applied to analyze the magnetizing field which is generated by a capacitor discharge impulse magnetizing fixture.**

*Index Terms***—Anisotropic bonded NdFeB permanent magnet, magnetizing analysis, vector Jiles-Atherton hysteresis model.** 

## I. INTRODUCTION

Bonded NdFeB permanent magnets based on anisotropic magnet powders are attractive for many applications since they offer significant advantages in terms of flexibility of manufacture compared to sintered one, while having relatively higher residual magnetic flux density and maximum magnetic energy product than conventional *isotropic bonded* NdFeB permanent magnets [1].

In order to improve magnetic performance as much as possible, it is necessary to ensure that the anisotropic magnet powders are fully aligned with the proper orientation during forming process.

Although the magnetic property of the *anisotropic bonded* NdFeB magnets such as residual magnetic flux density mainly depends on the orientation ratio of particles which is decided by alignment field, there does not exist any general guidance for magnetizing the *anisotropic bonded* NdFeB magnet.

Scalar Jiles-Atherton hysteresis model has been already applied to predict the residual magnetic flux density distribution [2]. However, it is not accurate for anisotropic magnet because, in fact, not only easy axis but also hard axis has contribution to magnetization *M* of PM, especially when the magnetic induction vector is parallel with the hard axis of the magnet. Therefore, it is necessary to utilize the vector J-A hysteresis model to predict residual magnetic flux density accurately.

In this paper, a systematic numerical method which combines transient finite element method (FEM) with vector J-A hysteresis model is proposed to predict the distribution of the residual magnetic flux density of the *anisotropic bonded* NdFeB magnet accurately.

## II. PROBLEM DESCRIPTION

In order to determine the residual magnetic flux density distribution, a transient FEM combined with scalar J-A hysteresis model is applied in our previous work. Fig. 1 shows the inner surface magnetic flux density distribution of PM with back yoke by using the previous presented method. From the figure, it can be said that the significant error between predicted and measured results happened at transition region of north and south poles. This is because in the transition region there is relatively bigger angle difference between easy axis of the anisotropic magnet and magnetizing field orientation.

In the previous work, we assume only the component of magnetizing field which is parallel with easy axis of PM contributed to magnetizing PM. However, in fact, even at hard axis of PM, the contribution of magnetization cannot be ignored, especially when big magnetizing field component is parallel with hard axis. As shown in figure 2, it can be seen that even when the angle difference between aligning direction and magnetizing direction is 90°, the magnetization is almost 10% compared with parallel condition. This component of M cannot be ignored when the accurate result is required. Therefore, a vector J-A hysteresis model combined FEM method is necessary to develop to estimate accurate residual magnetic flux density result for the anisotropic bonded NdFeB magnet.



Fig. 1. Calculated inner surface magnetic flux density distribution compared with measured one.



Magnetization tield (1esla)<br>Fig. 2. Normalized magnetization with different magnetization angle against preferred orientation (easy axis).

### III. VECTOR JILES-ATHERTON HYSTERESIS MODEL

The vector generalization process of the J-A hysteresis model is presented in [3]. In order to combine the vector J-A model with FEM, the inverse J-A model which takes the magnetic flux density as independent variable is defined. The detailed form of the inverse vector J-A model for 2-D implementation is given by [4].

In the main equations of the vectorized hysteresis model, an auxiliary vector variable was introduced:

$$
\vec{\chi}_f = \ddot{k}^{-1} \cdot (\mathbf{M}_{an} - \mathbf{M}_{ir}) \tag{1}
$$

where  $M_{an}$ ,  $M_{irr}$ , and  $\ddot{k}$  are the anhysteretic magnetization, irreversible magnetization, and a second rank tenser, respectively. The coefficients of the second rank tenser can be identified from measuring hysteresis loops of easy and hard axis of PM. The effective magnetic field vector variation is defined as follows:

$$
d\mathbf{H}_{\rm e} = d\mathbf{H} + \ddot{\alpha}d\mathbf{M} \tag{2}
$$

where **H** is the applied field and  $\ddot{\alpha}$  is a tenser which is identified from measurement data.

The evolution of the magnetization vector is calculated with vectorized hysteresis model accordingly to the sign of the scalar product between  $\vec{\chi}_f$  and dH<sub>e</sub> as follows:

1) If 
$$
(\vec{\chi}_f d\mathbf{H}_e) > 0
$$
  
\n
$$
d\mathbf{M} = \frac{1}{\mu_0} \left[ 1 + \vec{\chi}_f |\vec{\chi}_f|^{-1} \vec{\chi}_f (1 - \vec{\alpha}) + \ddot{c} \ddot{\xi} (1 - \dot{\alpha}) \right]^{-1} \cdot \left[ \ddot{\vec{\chi}}_f |\vec{\chi}_f|^{-1} \vec{\chi}_f + \ddot{c} \ddot{\xi} \right] d\mathbf{B}
$$
\n(3)

2) If 
$$
(\vec{\chi}_f d\mathbf{H}_e) \le 0
$$
  
\n
$$
d\mathbf{M} = \frac{1}{\mu_0} \Big[ 1 + \ddot{c}\ddot{\xi} (1 - \ddot{\alpha}) \Big]^{-1} \cdot \Big[ \ddot{c}\ddot{\xi} \Big] d\mathbf{B}
$$
\n(4)

where 1 is the diagonal unity matrix,  $\ddot{c}$  is a tenser calculated from experimental data, and  $\ddot{\xi}$  is the diagonal matrix of the derivatives of the anhysteretic functions with respect to the effective field component. The equations (3) and (4) are the vector forms of the inverse J-A hysteresis model and vector **M** can be derived from vector induction value **B**.

## IV. ANALYSIS METHOD

It is assumed that the permanent magnet material is orthogonally anisotropic [5]. Fig. 3 shows the analyzed capacitor discharge impulse magnetizer. The effects of eddy currents in the magnet can be ignored due to high resistivity of magnet. In 2-D analysis, the magnetic field governing equation in the FEM is written as:

$$
\nu \nabla \times \nabla \times \vec{A} = \vec{J}_0 + \nu_0 \nabla \times \vec{M}
$$
 (5)

where *v* is the medium reluctivity,  $v_0$  is the vacuum reluctivity, *A* is the magnetic vector potential,  $J_0$  is the applied current density calculated from the electric circuit, and *M* is the magnetization of permanent magnet determined from the initial magnetization curves or modeled hysteresis loops.

When a voltage source consisting of a capacitor is applied to the magnetizing winding, the discharge current is unknown. In this case, to calculate the magnetic field, it is necessary to couple (5) with the external circuit equation shown as follows:

$$
d\lambda/dt + RI + L_0 \, dI/dt - Q_0/C = 0 \tag{6}
$$

where  $\lambda$  is the flux linkage of the exciting coil for magnetizer,  $R$  is the winding resistance,  $L_0$  is the leakage inductance, and  $Q<sub>0</sub>$  is the initial charge stored in the capacitor. The discharge current *I* can be expressed by voltage on capacitor  $V_c$  and capacitance *C* as follows:

$$
I = -C dV_c/dt. \tag{7}
$$

In the analysis, the magnetization component  $M_{\rm v}$  and  $M_{\rm v}$  of PM in each element are calculated by vector J-A hysteresis model [6]. When the discharge current decreases to zero, *M* in each element of the PM is recorded. Finally, the residual *M* or *B* vector of the PM can be estimated. Fig. 4 shows the residual magnetic flux density vector distribution of PM. In the full version paper, the detail numerical analysis procedure and results will be shown.

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Fig. 3. Analysis model of magnetizer.



Fig. 4. *Br* vector distribution in PM.