Temperature Dependent Vector Hysteresis Model for Permanent Magnets

A.Bergqvist, D. Lin, and P. Zhou
Ansys Inc., Vestagatan 2D, Gothenburg SE-41664, Sweden
anders.bergqvist@ansys.com

Abstract — A hysteresis model for thermal and magnetic demagnetization of permanent magnets is proposed. It is based on the assumption that the evolution of the magnetization is driven by minimization of an energy function but is prevented from reaching the global minimum by a friction-like counterforce. Irreversible thermal effects resembling experiments emerge automatically by allowing the energy and the frictional force to be functions of temperature. The resulting model is computationally simple in any number of dimensions and all adjustable material parameters are easily found from measurements commonly available from permanent magnet suppliers.

I. INTRODUCTION

Permanent magnets are employed in various electromagnetic applications such as PM machines. Such devices can be subjected to conditions that cause partial demagnetization of permanent magnets, leading to a degraded performance. The demagnetization can come from high temperatures or from magnetic fields opposed to or perpendicular to the magnetization direction. Modelling such effects requires a temperature dependent hysteresis model. Inclusion of thermal effects has been proposed for the Jiles-Atherton model [1] and the Preisach model [2]. Maugin has suggested following the formalism of structural hysteresis models [3] which are often expressed in a thermodynamic framework where the incorporation of irreversible thermal effects is nearly automatic. In this work, we will formulate such a model for magnetics and include thermal effects with a minimum of additional assumptions and adjustable parameters.

II. MODEL FORMULATION

We begin with the case without thermal effects. basically using a simplified case of the model presented in [4], briefly outlined here. According to the principle of energy conservation, change in work W equals the sum of change in free energy F and loss Q,

$$\dot{W} = \dot{F} + \dot{Q} \tag{1}$$

Consider these three terms in turn. The rate of work is $\dot{W} = \mu_0 \mathbf{H} \cdot \dot{\mathbf{M}}$. We assume that the free energy F of the material depends only on the current magnetization $\mathbf{M}$ so that $\dot{F} = (\partial F / \partial \mathbf{M}) \cdot \dot{\mathbf{M}}$. Furthermore assume that the loss is proportional to the absolute value of $\dot{\mathbf{M}}$ by some coefficient $\mu_0 k$, giving

$$\mathbf{H} \cdot \dot{\mathbf{M}} = \mathbf{H}_{\mathrm{an}} \cdot \dot{\mathbf{M}} + k\,|\dot{\mathbf{M}}| \tag{2}$$

where we also have defined $\mathbf{H}_{\mathrm{an}}(\mathbf{M}) = (\partial F/\partial \mathbf{M})/\mu_0$. Quasistatic hysteresis appears due to using $|\dot{\mathbf{M}}|$ in the loss term since it means that for a given $\mathbf{H}$, there are multiple values of $\mathbf{M}$ such that (2) is satisfied and vice versa. In the loss-free case with $k = 0$, (2) reduces to $\mathbf{H} = \mathbf{H}_{\mathrm{an}}(\mathbf{M})$, which justifies considering $\mathbf{H}_{\mathrm{an}}(\mathbf{M})$ as the anhysteretic curve. We may interpret the behavior as follows: The magnetization strives towards a value such that $G(\mathbf{H},\mathbf{M})=F(\mathbf{M}) - \mu_0\, \mathbf{H}\cdot\mathbf{M}$ is minimized. The global minimum occurs when $\mu_0\, \partial G/\partial \mathbf{M} = \mathbf{H}_{\mathrm{an}}(\mathbf{M}) - \mathbf{H} = 0$. However all values such that $|\mathbf{H} - \mathbf{H}_{\mathrm{an}}(\mathbf{M})| \le k$ are metastable due to a friction-like resistance. Should $\mathbf{H}$ attain a value such that $|\mathbf{H} - \mathbf{H}_{\mathrm{an}}(\mathbf{M})| > k$, then $\mathbf{M}$ will instantly move in a direction parallel to $(\partial G/\partial \mathbf{M})$ and by such an amount that the state $|\mathbf{H} - \mathbf{H}_{\mathrm{an}}(\mathbf{M})| = k$ is reached. The model is basically a variation of Coulomb friction, a very old vector hysteresis model.

Numerically, the evolution law for getting $\mathbf{M}$ from $\mathbf{H}$ can be expressed as a two-step process. We employ the anhysteretic field $\mathbf{h}_{\mathrm{a}} := \mathbf{H}_{\mathrm{an}}(\mathbf{M})$ as an auxiliary variable. As shown in [4], the relation between $\mathbf{H}$ and $\mathbf{h}_{\mathrm{a}}$ is that of a vectorial play and an incremental change in $\mathbf{h}_{\mathrm{a}}$ can be expressed

$$\Delta \mathbf{h}_{\mathrm{a}} = \left(1 - \min\left(\frac{k}{|\mathbf{H} - \mathbf{h}_{\mathrm{a}}|}, 1\right)\right)(\mathbf{H} - \mathbf{h}_{\mathrm{a}}) \tag{3}$$

The magnetization is then found as

$$\mathbf{M} = \mathbf{M}_{\mathrm{an}}(\mathbf{h}_{\mathrm{a}}) \tag{4}$$

where $\mathbf{M}_{\mathrm{an}}$ is the inverse function of $\mathbf{H}_{\mathrm{an}}$.

We then turn to the case where the temperature T is allowed to change. Temperature will obviously affect the energy F in the material. As k can be interpreted as a manifestation of a small ripple contribution to energy [4], this too can be expected to be a function of temperature. A straightforward generalization of the temperature independent model is then to simply allow F, and by implication $\mathbf{M}_{\mathrm{an}}$, and k to be functions of temperature. Thus we generalize the evolution equations (3),(4) to

$$\Delta \mathbf{h}_{\mathrm{a}} = \left(1 - \min\left(\frac{k(T)}{|\mathbf{H} - \mathbf{h}_{\mathrm{a}}|}, 1\right)\right)(\mathbf{H} - \mathbf{h}_{\mathrm{a}}) \tag{5}$$

$$\mathbf{M} = \mathbf{M}_{\mathrm{an}}(\mathbf{h}_{\mathrm{a}}, T) \tag{6}$$

Expressions (5),(6) may be used to evaluate $\mathbf{M}$ for any arbitrary variations of $\mathbf{H}$ and/or T over time.

III. PARAMETER DETERMINATION

The adjustable material parameters in the model are the functions $\mathbf{M}_{an}(\mathbf{h}_a,T)$ and $k(T)$. We shall here only consider the isotropic case. Consider a demagnetization curve at some given temperature T. Along this curve, it holds that $h_a = H + k(T)$, so

$$M_{\mathrm{demag}}(H,T) = M_{\mathrm{an}}(H + k(T), T) \tag{7}$$

Due to symmetry, $M_{an}(0,T)$=0. Thus, $k(T)$ is the intrinsic coercivity at temperature T and $M_{an}(H,T)$ is found by shifting the demagnetization curve by $k(T)$ and so all adjustable parameters can be found from a set of demagnetization curves measured under different temperatures. Such data is often provided by manufacturers of permanent magnets. Furthermore, a parametric model for such curves was proposed in [5]. This result can be incorporated here, in the simplest case leading to

$$k(T) = k(T_0)[1 + \beta(T - T_0)] \tag{8}$$

$$M_{\mathrm{an}}(H,T) = M_{\mathrm{an}}(H,T_0)[1 + \alpha(T - \mathrm{T}_0)] \tag{7}$$

Here T_0 is some reference temperature for which a demagnetization curve is measured and α,β are adjustable parameters giving a linear dependence in temperature for coercivity and remanence respectively. All adjustable parameters can in that case be found from a demagnetization curve measured at one temperature, and intrinsic coercivity and remanence measured at another temperature.

IV. SOME RESULTS

Some typical results are shown in Figs 1-2 using $M_{an}(H,T_0)= \chi H/(1+\chi|H|/M_s)$ with M_s=1.0 MA/m, χ=100 and thermal coefficients α=0.001K^{-1}, β =0.005K^{-1}. Isothermal major M-H loops are shown in Fig 1. Minor loops consist of horizontal segments crossing between the major loop branches. Real materials exhibit more complex minor loops. This is a limitation due to the assumption that the energy and the loss depend only on $\mathbf{M}$ and $|\dot{\mathbf{M}}|$ respectively.

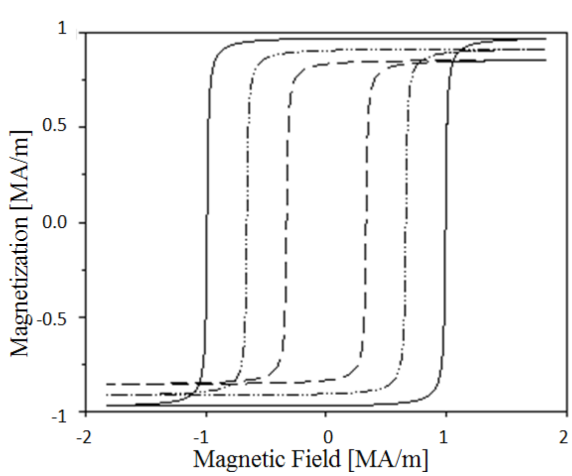

Fig. 1. Magnetization vs field strength at T=0 °C, 60 °C and 120 °C.

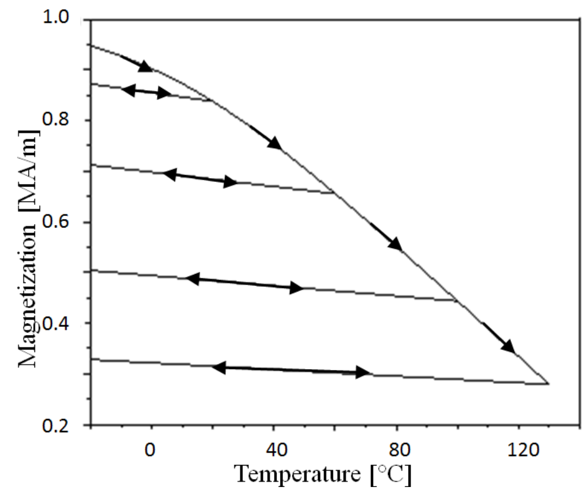

Fig. 2. Thermal demagnetization in magnetic circuit with airgap.

Fig 2. shows an example of thermal demagnetization in a magnetic circuit with airgap. After initial magnetization, the temperature is repeatedly increased and decreased with gradually higher peak value. Here neither H or M is known a priori but they are determined through a constant ratio between H and B. Such experiments were performed in [6] and these calculations show very similar behavior.

V. CONCLUSIONS

A hysteresis model has been presented that predicts magnetization for arbitrary variations in field and temperature. It is based on a thermodynamic formulation where irreversible thermal effects emerge automatically and no adjustable parameters appear other than those found from isothermal measurements. Thermal demagnetization results resemble those of experiments. Limitations that will be addressed in future work are that minor loop behavior is overly simplistic and that thermal demagnetization and demagnetization from field cycling lead to the same state.

VI. REFERENCES

[1] A. Raghunathan, Y. Melikhov, J. E. Snyder, D. C. Jiles , “Theoretical Model of Temperature Dependence of Hysteresis Based on Mean Field Theory”, ”, *IEEE Trans. Mag.*, Vol 46, no 6, pp. 1507-1510, 2010.

[2] G. Bertotti, V. Basso, M. LoBue, A. Magni: Thermodynamics, hysteresis, and thermal relaxation in: The Science of Hysteresis, editors Giorgio Bertotti, Isaak D. Mayergoyz, Academic Press (2006), ISBN 0124808743

[3] G.A.Maugin, “Compatibility of Magnetic Hysteresis with Thermodynamics”, , *Int J. Appl. Electromag. mat.* no. 2, pp. 7-19, 1991.

[4] A. Bergqvist “Magnetic Vector Hysteresis Model with Dry Friction-like pinning”, *Physica B*, Vol 233, no 4, pp. 342-347, 1997.

[5] P. Zhou, D. Lin, Y. Xiao, N. Lambert, M.A. Rahman, “Temperature-Dependent Demagnetization Model of Permanent Magnets for Finite Element Analysis”, *IEEE Trans. Mag.*, Vol 48, no 2, pp. 1031-1034, 2012.

[6] Y. Kato, “Thermal stability of sintered and bonded rare-earth magnets”, *J. Appl. Physics*, Vol 85, no. 8, pp.4868-4870, 1999.