Modeling of frequency selective surfaces using impedance type boundary condition

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***Abstract*—The electromagnetic properties of frequency selective surfaces (FSS) are often calculated using integral equations. The metal strips of an FSS are usually modeled based on the assumption that the electric field is constant in the metal with respect to its depth. This assumption is valid as long as the thickness of the metal strip is significantly shorter than the skin depth. If this condition is not applicable, the discretization of the volume of the metal strip is necessary. In this paper we propose to use the impedance type boundary condition to avoid the volumetric discretization when the thickness of the metal strip is comparable to the skin depth. We will show that using the proposed method the accuracy of the modeling is increased while the number of unknowns remains very low compared to the full volumetric discretization.**

***Index Terms*—Frequency selective surfaces, Impedance type boundary conditions, Integral equations**

I. Introduction

Frequency selective surfaces (FSS) are periodic metallic structures formed on a dielectric layer. They are often used to filter the electromagnetic waves with respect to the frequency or to the angle of incidence. Some possible FSS structures are shown in Fig. 1 [1]. The thickness of the metal is usually considerably smaller than the wavelength of the electromagnetic field around, while the periodicity of the metallic pattern is in the range of the wavelength. For the design of such FSS, fast analysis methods are required, that is why it is worth to consider numerically cheap approximate methods.

In this paper first we consider the traditional method using integral equations when the metallic parts are modeled as surface currents [2], [3]. This method is very efficient, however it might fail when the thicknesses of the metallic parts are comparable to the skin depth. In such cases the volumetric discretization of the metallic parts are needed to obtain accurate results. In this contribution we propose an approximate method based on the impedance type boundary condition for the cases when the thicknesses of the metallic parts are in the range of one to several skin depths. This method can serve as a trade-off between accuracy and numerical expenses. In the following we describe the proposed method and compare its results to those obtained by the traditional method and by a rigorous finite element method (FEM) analysis.

This work is supported by the Grant TAMOP-4.2.2.B-10/1-2010-0009 and by the Hungarian Scientific Research Fund under Grant K-105996

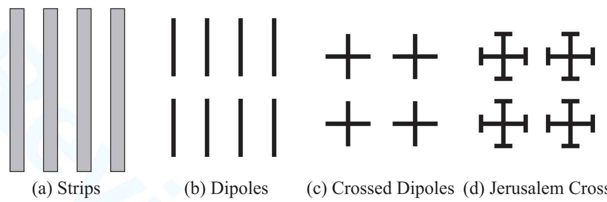

Figure 1: Examples for FSS arrangements

II. Calculation methods based on integral equations

A. Traditional method: metal as surface current

Consider the configuration shown in Fig. 2. Here we can see a conductor that is deposited on a surface separating two dielectric materials with permittivities ε_1 and ε_2. The thickness of the conductor is d, the surface S occupied by the metal in the xy plane is independent of z when $0 < z < d$. The conductivity and the permeability of the conductor is σ and μ_0, respectively.

The electric field $\vec{E} = \vec{E}^i + \vec{E}^s$ can be obtained as the sum of an incident ($\vec{E}^i$) and a scattered ($\vec{E}^s$) fields. $\vec{E}^s$ is generated by the currents $\vec{J}$ flowing in the metal. This can be written using the dyadic Green's functions $\mathbf{G}(x, y, z|x', y', z')$ transforming the electric current distribution into the electric field [4] as,

$$\vec{E}^s = \int_V \mathbf{G}(x, y, z|x', y', z') \cdot \vec{J}(x', y', z')dV'. \qquad (1)$$

If the $\lambda \gg d$, where $\lambda = \frac{2\pi c}{\omega\sqrt{\varepsilon_1}}$ is the wavelength of the electromagnetic field in the $z > 0$ region (c is the speed of light in vacuum and ω is the angular frequency of the excitation) we may use the following approximation,

$$\mathbf{G}(x, y, z|x', y', z') \approx \mathbf{G}(x, y, z|x', y', z' = 0) = \mathbf{G}_0(x, y, z|x', y'), \quad 0 < z' < d. \qquad (2)$$

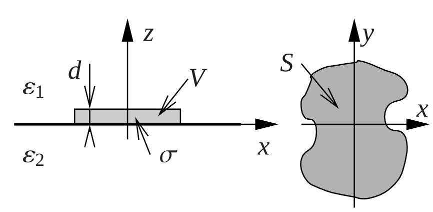

Figure 2: Schematic drawing of the investigated arrangement

If the thickness of the metal $d \ll \delta$ (where $\delta = \sqrt{\frac{2}{\omega\sigma\mu_0}}$ is the skin depth in the metal), the current density can be considered to be homogeneous with respect to the z coordinate direction, i.e. $\vec{J}(x, y, z) \approx \vec{J}_0(x, y)$ where $\vec{J}_0$ lies in the xy plan. Using (2) and this assumption, (1) is simplified as,

$$\vec{E}^s = d \int_S \mathbf{G}_0(x, y, z|x', y') \cdot \vec{J}_0(x', y') dS'. \qquad (3)$$

By introducing the surface current density $\vec{J}_s(x, y) = \vec{J}_0(x, y)d$ that is assumed to be concentrated on the $z = 0$ surface, and understanding that the continuity of $\vec{E}_t$ (subscript t stands for the tangential component of the vector) at the metal interfaces implies that $\sigma d \left(\vec{E}_t^s + \vec{E}_t^i \right) = \vec{J}_s$ we can arrive to the following integral equation,

$$\vec{J}_s = \sigma d \vec{E}_t^i + \hat{n} \times \left[\sigma d \int_S \mathbf{G}_0 \cdot \vec{J}_s dS' \right] \times \hat{n}, \quad x, y \in S, \qquad (4)$$

where $\hat{n}$ is the normal vector of the surface of the metal. The solution of (4) (done by, e.g., the Method of Moment) will provide the unknown surface current distribution $\vec{J}_s$, consequently the electromagnetic field can be obtained in the whole arrangement.

B. Proposed method: metal as impedance type boundary condition

Here we approximate the currents inside the metal as,

$$\vec{J}(x, y, z) \approx \vec{J}^+(x, y)e^{-\gamma z} + \vec{J}^-(x, y)e^{\gamma z}, \qquad (5)$$

where $\vec{J}^+$ and $\vec{J}^-$ lie in the xy plan and $\gamma = \sqrt{j\omega\sigma\mu_0}$. This assumption can be accepted as long as the dimensions of the metal in the xy plane is considerably larger than the skin depth δ. Note that –contrary to the case shown above– the thickness of the conductor might be in the range of the skin depth.

Taking again the approximation (2) and substituting this and (5) into (1), the integration with respect to z' can be evaluate analytically. We will obtain,

$$\vec{E}^s = \int_S \mathbf{G}_0 \cdot \left[\frac{1 - e^{-\gamma d}}{\gamma} \vec{J}^+(x', y') + \frac{e^{\gamma d} - 1}{\gamma} \vec{J}^-(x', y') \right] dS'. \qquad (6)$$

Having this approximate expression and considering the continuity of $\vec{E}_t$ at the top ($z = d$) and at the bottom ($z = 0$) surfaces of the metal, we can calculate the currents as,

$$\sigma \vec{E}_t \,|_{z=0} = \vec{J}^+ + \vec{J}^- = \sigma \vec{E}_t^i \,|_{z=0} + \sigma \vec{E}_t^s \,|_{z=0}, \quad x, y \in S, \qquad (7)$$

$$\sigma \vec{E}_t \,|_{z=d} = \vec{J}^+ e^{-\gamma d} + \vec{J}^- e^{\gamma d} = \sigma \vec{E}_t^i \,|_{z=d} + \sigma \vec{E}_t^s \,|_{z=d}, \; x, y \in S. \qquad (8)$$

If we put the expression (6) into (7) and (8), we obtain two coupled integral equations that can be solved for the unknowns $\vec{J}^+(x, y)$ and $\vec{J}^-(x, y)$. The integral equations are solved using the Method of Moment, consequently the electromagnetic field can be described in the whole arrangement.

If we assume the $z = d$ and $z = 0$ planes like the $z + 0$ and $z - 0$ sides of the $z = 0$ interface, than (7) and (8) are similar to the application of the impedance type boundary condition [5], [6] in the regions of the $z = 0$ plane where metal is deposited.

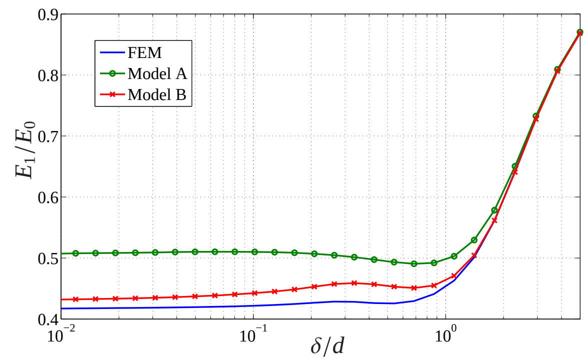

Figure 3: Ratio of the illuminating (E_0) and transmitted (E_1) fields for the three methods. FEM: reference solution, Model A: traditional method discussed in II-A, Model B: proposed method described in II-B

III. Numerical results

Numerical results obtained by the two methods described above are compared for the 2D geometry shown in Fig. 1(a). The widths, the period and the thickness of the metal stripes are 2 cm, 5 cm and 2 mm, respectively. The relative permittivity of the regions above and below the metal strips are assumed to be 1. The excitation ($\vec{E}^i$) is an x-polarized plane wave traveling to the $-z$ direction, the frequency of this is 3 GHz (the wavelength is $\lambda = 10$ cm). The conductivity of the metal is varied to simulate metals with different skin depths. As a reference solution, rigorous analysis of the configuration by FEM is carried out using the full discretization of the metal.

In Fig. 3 the ratio of the electric fields of the illuminating plan wave (E_0) and the x component of the electric field transmitted far to the region $z < 0$ (E_1) are shown. One can see that in the case when $d < \delta/2$ ($\delta/d > 2$) all methods provides the same result. However, for the cases when $d > \delta/2$ the modeling of the metals with surface currents gives poor result while the proposed method still provides acceptable approximation of the transmitted field.

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