

Homogenization of Periodic Structures Using the Finite Element Method

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Abstract—Homogenization is an efficient procedure to simulate multi-scale problems using the finite element method. Three different forms of homogenization are available to model small scale electromagnetic fields: a homogeneous sheet isotropic/anisotropic impedance boundary condition, a thin dielectric slab made of homogenous effective permittivity and permeability and a general 3D isotropic/anisotropic homogeneous material. This paper gives an overview of homogenization including new improvements to the three procedures using the finite element solution of a unit cell. Applications are presented to demonstrate the efficiency of the method.

Index Terms—Multiscale problems, Homogenization, Finite Element Method (FEM), Metamaterials, Periodic structures, Frequency Selective Surface (FSS)

I. INTRODUCTION

Periodic sub-structures are often used in RF and Microwave devices and antennas to reduce size and improve functionality. While the size of the device or antenna is on the order of a wavelength or larger, the size of the sub-structure is fractions of a wavelength. The resulting complexity of these devices makes direct simulation of these structures beyond current hardware capabilities. Even advanced methods such as the Domain Decomposition Method (DDM) can't cope with simulating such structures as tri-band satellite antenna systems containing large screens of fine periodic structures. To overcome this sub-wavelength geometric detail problem, complex periodic substructures can be replaced by thin sheets having an equivalent impedance boundary condition [1]. This procedure is called homogenization. Other approaches to homogenization are to use a dielectric slab of finite thickness or a 3D material with a uniform isotropic or anisotropic effective permittivity and/or permeability [2],[3]. Homogenization greatly reduces the computer resources required in the FEM simulation. It is clear that homogenization is effective only if the homogenization properties of the periodic structure are accurately determined. This paper introduces new extraction techniques for computing the homogenization properties by analyzing a unit cell of the periodic structure using the FEM and applying new post processing techniques.

II. HOMOGENIZATION METHODS

A. Effective anisotropic sheet impedance boundary condition

When the thickness of the periodic sub-structure is much smaller than a wavelength, the geometric complexity of the periodic sub-structure can be replaced by an anisotropic sheet impedance boundary condition. This equivalent anisotropic

sheet impedance takes the polarization of the incident field into account. Since the FEM enforces the continuity of the tangential components of the electric field, these components will be continuous across the impedance boundary. This equivalent sheet impedance method is discussed in [1] stressing that it is valid in the quasi-stationary limit when the wavelength is much longer than the lattice of the structure. However, investigations reveal that, in practice, this ratio can be smaller than the stationary limit. The key issue is that the reflection and transmission coefficients of the layer should be relatively independent on the incident angle of the incident field. To demonstrate this, a unit cell of a thin split ring resonator FSS [3] was analyzed and replaced by a large, anisotropic impedance sheet. Figure 1 shows that even with a relatively large wave length lattice ratio of 4.26, the S parameters of the unit cell and the homogenized large sheet are almost identical.

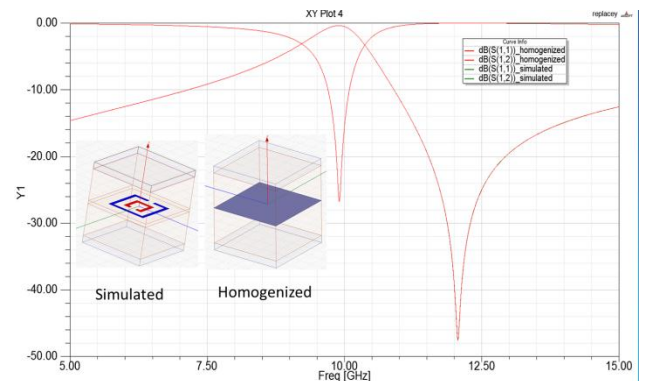


Figure 1. S parameters of a split ring resonator. Simulated unit cell and homogenized solutions

B. Equivalent thin dielectric slab

When the physical thickness of the structure is large, the sheet impedance approximation cannot be applied since that method enforces the tangential continuity of the E field across the sheet, which is no longer valid. In this case, an equivalent dielectric slab can be used for homogenization. Figure 2 shows the dielectric slab, which is treated as a transmission line.

The required physical parameters of the dielectric slab are the input short and open circuit impedances (Z_{short} Z_{open}). These can be determined from the unit cell model of the FSS using open and short circuit load conditions. These loads can be implemented with Perfect Electric Conductor (PEC) and Perfect Magnetic Conductor (PMC) boundary conditions, respectively. Knowing these measured or calculated

impedances, the characteristic wave impedance Z_{od} and propagation coefficient γ_d are determined as

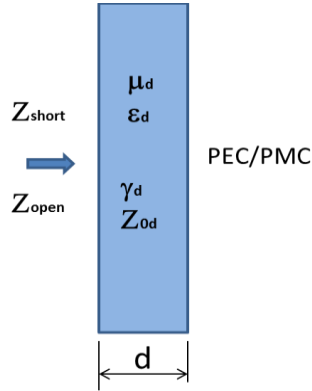


Figure 2. Dielectric slab as a transmission line model

$$Z_{od} = \sqrt{Z_{short} Z_{open}} ; \quad \gamma_d = \frac{1}{2d} \ln(a^2) \quad (1)$$

where

$$Z_{short} = Z_{od} \operatorname{th}(\gamma_d d) ; \quad a^2 = -\frac{Z_{short} + Z_{od}}{Z_{short} - Z_{od}} \quad (2)$$

and d is the thickness of the slab and $\operatorname{th}()$ means hyperbolic tangent function. The material properties of the slab are

$$\epsilon_d = j \frac{\gamma_d}{\omega Z_{od}} ; \quad \mu_d = \frac{j Z_{od} \gamma_d}{\omega} \quad (3)$$

The literature contains many discussions regarding the uniqueness of the model, since the phase of the complex logarithmic function is ambiguous [3], [4]. If the thickness of the slab is smaller than the quarter wavelength, one can assume the zero order phase value for γ_d . The method will be extended to oblique incidence, too.

C. Effective 3D material

The most general case is when a complex 3D periodic sub-structure has to be homogenized. In this case, a 3D anisotropic effective material has to be used. The x component of the permeability can be calculated by averaging the magnetic field

$$\mu_{effx} = \frac{B_{avex}}{\mu_o H_{avex}} \quad (4)$$

where

$$B_{avex} = a^{-2} \int_{S_{yz}} \mu_o \mathbf{H} \cdot \mathbf{nds} ; \quad H_{avex} = a^{-1} \int_{C_x} \mu_o \mathbf{H} \cdot \mathbf{dc} \quad (5)$$

Here S_{yz} and C_x are a surface of z normal and a curve of x tangential, respectively.

Once again, the question is: Is the model limited to the quasi-stationary limit or does it apply to higher frequencies? Pendry [5] uses just one side surface of the unit cell for the surface integral and one line parallel to the x coordinate for the line integral. Investigations show that the quasi-stationary limit of this choice is very high, $\lambda/a > 30$. Amert [4] uses a more advance averaging method, which is improved in this paper.

Figure 3 shows the effective permeability of a 3D split ring resonator structure [5],[7] using a new averaging method with $\lambda/a = 5$. This is nearer to the quasi-stationary limit and beyond the limit of the old method. Consequently, the results below are slightly different from those in [6].

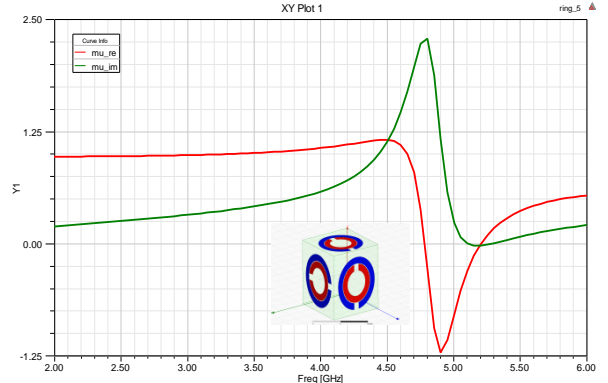


Figure 3. Real and imaginary part of the effective permeability of a 3D split ring resonator.

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