A Paretian Approach to Optimal Design with Uncertainties: Application in Induction Heating

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***Abstract*—A major issue in optimal design of electromagnetic devices relates to optimizing against uncertainty, in terms of geometric and physical parameters. The induction heating of a graphite disk, with the purpose to obtain a prescribed temperature profile, is considered as the model problem. The novelty of the paper is a cost-effective method enabling the designer to select the Pareto optimal solutions trading off design criterion and sensitivity.**

***Index Terms*— Multiphysics field problems, finite elements, multiobjective optimal design, design uncertainties.**

I. Introduction

A major issue in optimal design of electromagnetic devices relates to optimizing against uncertainty. In fact, when a physical device or component is modelled, there is always some degree of uncertainty associated with all real parameters, like *e.g.* geometric quantities or material properties. As a consequence, tolerance intervals are defined on almost all values: therefore, a real design system needs to find an optimal result which has a performance as insensitive as possible against small parameter changes. Actually, this is a secondary design criterion, in addition to the performance criterion: a bi-objective optimisation problem is so originated, the most general solution of which is represented by the relevant Pareto front. There are several contributions on the topic of uncertain design or identification with uncertainties, see *e.g.* [1-4]; a comparative review of optimization procedures based on worst case scenario was announced in [5], while in [6] an approach based on the approximated Lipschitz constant was considered. Usually, a generalized error functional, incorporating both the design criterion and its sensitivity, is minimized so obtaining a unique solution which is assumed to be the optimum. In the paper, the optimal design problem with uncertain parameters is formulated and solved in terms of Paretian optimality, indentifying a set of optimal solutions characterized by a different degree of sensitivity. The induction heating of a graphite disk, with the purpose to obtain a prescribed temperature profile, is considered as the model problem.

II. Optimal design method

Having defined the n_v-dimensional vector $\boldsymbol{g}$ of unknown design variables, and the n_p-dimensional vector $\boldsymbol{p}$ of uncertain parameters with the relevant uncertainty intervals, in general the design criterion $\boldsymbol{f}$ will depend on both $\boldsymbol{g}$ and $\boldsymbol{p}$. The problem reads: find the family of non-dominated solutions $\tilde{\boldsymbol{g}}$ minimizing the pair of objective functions $f_1 = f(\boldsymbol{g},\boldsymbol{p})$ and $f_2 = D_p f(\boldsymbol{g},\boldsymbol{p})$, subject to the problem constraints. In particular, $D_p f$ is the first-order sensitivity of the design criterion against small variations of the uncertain parameters around a given design point; it is estimated in a cost-effective way by means of a fractional design of experiments [7]. In turn, the Pareto front in the objective space (f_1,f_2) is approximated by means of a standard NSGA-II technique [8]. It can be underlined that the sensitivity is computed with respect to uncertain parameters, which are not design variables. Therefore, the use of a global oriented optimizer like NSGA, which acts on the design variables only, doesn't affect the sensitivity computation which is local information on a specific solution.

III. The model

Fig. 1 shows the geometry of the 2D axisymmetric model implemented [9-10]: the problem is to heat a graphite disk uniformly up to a given temperature (close to 1100°C) value using an inductor with 12 copper turns supplied by a 5 kHz (±15%) sinusoidal current with suitable amplitude in order to transfer the device a power of 60 kW. A steady state thermal problem coupled to a time-harmonic magnetic problem is solved [11]. A typical mesh of the magnetic model exhibits 128,000 nodes and 60,700 elements, whereas the thermal problem is solved on the disk region and is characterized by a mesh of approximately 6,000 nodes.

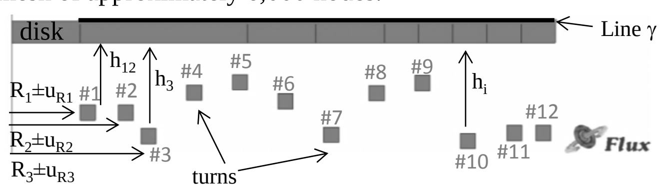

Fig. 1. Model geometry and design variables

A. *Inverse multiobjective problem*

Let us consider two vectors, $\boldsymbol{g}$ and $\boldsymbol{p}$, dependent on 10 design variables and 3 uncertainty parameters, respectively. Specifically, the vertical positions of turns are the design variables of vector $\boldsymbol{g}$, while three parameters are affected by uncertainty (vector $\boldsymbol{p}$). Accordingly three classes of optimization problem are considered: (a) three radial positions of turns are uncertain as shown in Fig. 1, (b) two uncertain parameters are in the physical domain (frequency, and graphite resistivity) and the third one in the geometric domain (#1 central turn radius), and, finally, (c) three uncertain parameters in the physical domain (frequency, and resistivity of graphite and copper). Table I reports the parameters, their nominal values and uncertainty range for the three problem classes considered. The uncertainty interval of the frequency relates mostly to uncertainty of the resonance circuits components.

TABLE I

UNCERTAIN PARAMETERS: NOMINAL VALUES AND UNCERTAINTY INTERVALS FOR THE THREE CLASSES OF PROBLEMS.

Problem 1	R_1 [mm]		R_2 [mm]		R_3 [mm]	
	45	±5	70	±5	95	±5
Problem 2	$\rho_{graphite}$ [10^{-6} Ωm]		frequency [Hz]		R_1 [mm]	
	10.25	±2.5	5000	±750	45	±5
Problem 3	$\rho_{graphite}$ [10^{-6} Ωm]		frequency [Hz]		ρ_{copper} [10^{-8}Ωm]	
	±2.5	±2.5	5000	±750	1.7	±0.1

The objective functions of the optimization problem are the disk temperature uniformity (f_1) and its sensitivity (f_2). The temperature uniformity (f_1) is evaluated on the line γ using the "criterion of proximity" defined in [10] and based on the counting of the number of points on the line γ (Fig.1) where the sampled temperature is close to a given value considering a tolerance interval of ±2.5°C. The quantity to be minimized is the number of points that do not satisfy the previous condition. Sensitivity (f_2) is computed using a design of experiments (DOE) strategy. For a given set of design variables, four additional solutions are computed by varying the parameter values according to the Table II [7]. The signs '+' and '-' correspond to choose the upper or lower limit on the uncertainty range of the given parameter in Table I, respectively. The set ($f_{n,1}$, $f_{n,2}$, $f_{n,3}$, $f_{n,4}$) around the current design vector is computed. The sensitivity is evaluated using the following strategy: for the parameter k, $k=1..3$, the sums of f_1 values corresponding to a '+' in Table II, $S_{+,Pk}$, and the ones corresponding to a '-', $S_{-,Pk}$, are computed. For each parameter the partial sensitivity, s_{Pk}, is evaluate as:

$$s_{Pk} = \frac{S_{+,Pk}}{N_+} - \frac{S_{-,Pk}}{N_-} \qquad (2)$$

where N+ and N- are the number of sign '+' and '-' in the column corresponding to the considered parameter in Table II. Finally, the total sensitivity, f_2, is given by:

$$f_2 = \frac{1}{\max(f_2)} \sqrt{\sum_{k=1}^{3} s_{Pk}^2} \qquad (3)$$

TABLE II

SIGN ALTERNANCE FOR DOE EVALUATION OF f_1

	P_1	P_2	P_3	f_1
Y_1	+	+	+	$u_{n,1}$
Y_2	-	+	-	$u_{n,2}$
Y_3	-	-	+	$u_{n,3}$
Y4	+	-	-	$u_{n,4}$

IV. RESULTS

Fig. 2 shows the Pareto front obtained by solving an optimization problem (a), whereas in Table II the values assumed by the design variables considering the best solution in terms of temperature uniformity and the ones in terms of lower sensitivity are reported. Fig. 3 shows the temperature along line γ for a few cases in Table III, whereas in Fig. 4 an example of the turn positions is shown.

TABLE III

BEST SOLUTIONS IN TERMS OF THERMAL UNIFORMITY AND SENSITIVITY.

	h_{1-2}	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	h_{11-12}	f_1	f_2
S_1	37.1	21.2	43.1	28.6	4.6	10.6	24.1	16.3	14.2	12.2	80	0.57
S_2	36.8	21.7	44.2	27.7	3.5	10.5	23.9	16.1	13.8	12.3	103	0.13

V. CONCLUSION

A cost-effective method enabling the designer to select the best solutions trading off design criterion and sensitivity has been proposed, considering both geometric and physical parameters as the source of uncertainties. In general the Pareto exhibits a dependence on both design criteria, thermal uniformity and sensitivity.

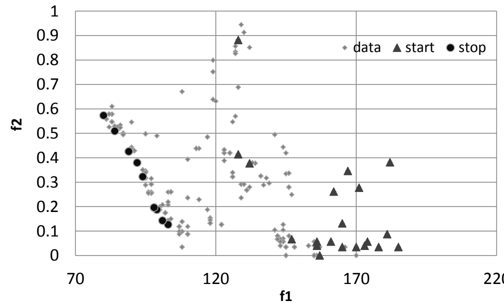

Fig. 2. Pareto front: f_1 temperature uniformity and f_2 sensitivity.

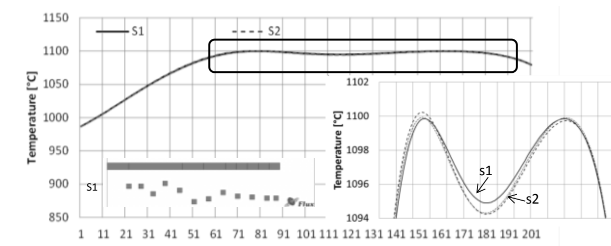

Fig. 3. Temperature along line γ for the best solutions in terms of uniformity (S_1) or sensitivity (S_2) and Turn positions for solution S_1.

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