Modal analysis of currents induced by Magnetic Resonance Imaging gradient coils

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***Abstract*—In magnetic resonance imaging (MRI) gradient coils are switched during fast current pulse sequences. These time-varying field interact with the conducting surround of the scanner producing deleterious effects such as imaging distortions and joule heating. Using a multi-layer integral method the spatio-temporal nature of the eddy currents induced by gradient coils is investigated. The existence of the Eigenmode is experimentally demonstrated be measuring the magnetic field and the time decay constant of a typical z-gradient coil and its interaction with a conductive cylinder. The method can be used to understand and mitigate undesired effects of eddy currents in MRI scanners.**

***Index Terms*—Eddy currents, integral equations.**

I. INTRODUCTION

Gradient coils produce a linear variation of the magnetic field axial component along the *x*, *y* and *z* coordinates to spatially localize the magnetic resonance imaging (MRI) signal. Long lasting eddy currents are invariably induced in the surrounding conducting structure when the gradient coils are rapidly switched during a pulsing time sequence. The eddy current in turn, generates a secondary magnetic field in the imaging region that opposes and spatially distorts the primary field produced by the gradient coils; resulting in undesired imaging distortions [1]. Others deleterious effects are of major concern when joule heating is transmitted to the sample or causes un-pleasant acoustic noise during imaging time [2]. Understanding the origin and how to mitigate the effects of eddy currents in MRI have been one of main point of interest of coils designer in MRI instrumentation area. To the best of the author knowledge there is one attempt to explain the relationship between the eigenmodes and eddy currents and the magnetic field produced by these currents in MRI [3]. This paper presents a detail eigenmode and eigenvalue analysis of eddy currents induced by MRI gradient coils and the experimental validation of the modes existence using a *z*-gradient coil situated in a hollow aluminum cylinder. A multi-layer integral method (MIM) is used to simulate the eddy currents and its corresponding eigenmodes and values[4].

II. METHOD

A. Theory

The MIM [4] assumes that a non-magnetic, conducting, thin volume region $\Omega \in \Re^3$ with a linear isotropic conductivity σ is immersed in a time-varying external magnetic field produced by a current source $\mathbf{J}_s(\mathbf{r},t)$. The conducting region Ω, where one of the linear dimensions is much smaller than the remaining dimensions, is segmented along its normal $\mathbf{n}$ into *N* uniform layers of thickness *h*. *h* is considered to be much smaller than the skin depth at the given frequency. The equation that rules the current diffusion process is deduced by applying the law of the conservation of energy considering the resistance of the thin volume Ω as a dissipative function, hence [4, 5]:

$$\frac{d}{dt}\left(e^{\boldsymbol{\lambda} t}\mathbf{U}^{-1}\boldsymbol{\psi}_i(t)\right) = -e^{\boldsymbol{\lambda} t}\mathbf{u}\frac{ds(t)}{dt} \quad , \qquad (1)$$

$$\mathbf{u} = \mathbf{U}^{-1}\mathbf{M}_{ii}^{-1}\mathbf{M}_{is} \text{ and } \mathbf{R}_{ii}\mathbf{U} = \mathbf{M}_{ii}\,\mathbf{U}\boldsymbol{\lambda}.$$

The square matrix $\mathbf{U}$ contains the eigenmodes or energy states that can be excited by the current density $\mathbf{J}_s(\mathbf{r},t)$. The diagonal of the matrix $\boldsymbol{\lambda}$ rules the time duration of the excited mode. $\mathbf{M}_{is}$ is a vector matrix that contains the mutual inductive coupling between the source and the conducting shells, $\mathbf{R}_{ii}$ and $\mathbf{M}_{ii}$ are the self-resistive and inductive coupling values of the conducting shells, respectively. s(t) is the temporal variation of the driving current. The vector $\boldsymbol{\psi}_i(t)$ contains the unknown amplitude of the stream function $\psi_i(t)$ corresponding to the N_e number of nodes forming the mesh. $\mathbf{u}$ is a vector containing the subset of eigenmodes excited by the coil.

B. Measuring the eigenmode and eigenvalue

A 34-turn *z*-gradient coil with a radius of 125.5mm, wire diameter of 1mm was designed using the Boundary Element Method (BEM) [6]. The z-gradient coil was excited with a 1.48A trapezoidal pulse (200µs rise time, 20.2ms pulse duration) to induce eddy currents in a 2.5mm thick cylindrical conductor with an inner radius of 175mm, overall length of 387mm and electrical conductivity of 32.26MSm^{-1}. The coil was driving with a relative long pulse in order to assure that no secondary field (field induced by the eddy currents) is presented at the end of the pulse [4]. We used a low-noise TMR STJ-220 magnetic field sensor with AL-05 signal conditioning (MicroMagnetics). The total field (primary+secondary) was measured at multiple axial positions in increments of Δz = 5mm along the +z-axis. The field amplitude and the time decay constant corresponding to each axial position were obtained by using a single exponential fitting.

III. Results and Discussions

A. Eigenmodes and eigenvalues analysis

Fig 1, shows some of the many eigenmodes calculated using Eq. (1). The eigenmodes and the time decay constants correspond to the conductive cylinder where the eddy currents were induced by the z-coil.

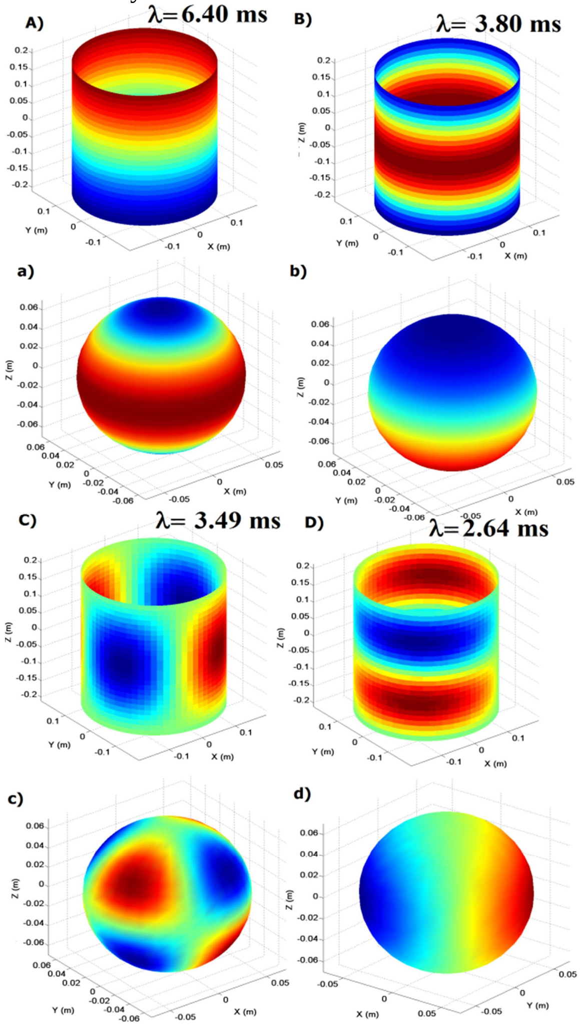

Fig.1 – **A**, **B** , **C**, **D**)Some of the many eigenmodes presented in the cylindrical conductive structure and the corresponding time decay constant. **a**, **b**, **c** and **d** are the magnetic field profile induced by the modes. Values given in arbitrary units.

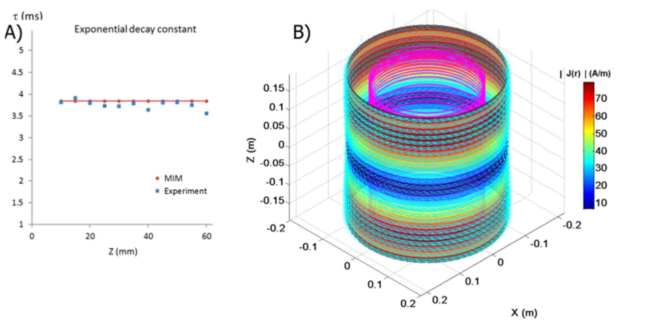

Fig.2 – **(A)** decay constant measured and predicted by MIM τ= 3.805 ms. **(B)** Absolute value of the induced current density predicted by MIM.

The convolution $e^{\lambda t}\mathbf{u}\frac{ds(t)}{dt}$dictates the amplitude, the time duration and which of the modes are excited by the coil. The mode **A**) produces a very homogenous field (**a**) and it is believe that is the main cause of the B_0-shift in spectroscopy and imaging[1]. The mode **B**) is generates a (**b**) gradient field along the z-coordinate. This mode is excited by coils with similar configuration such as z-gradient coils. The mode **D**) has a similar spatial profile of that presented in x/y-gradient coils hence a linear field (**d**) is produced along the x/y direction. This mode degenerates in two, but 45° respect each other. Gradient coil of complex geometry (such as shielded coils) may excite many modes resulting in a complex spatiotemporal superposition of all excited modes. Modes of small energy such as **D**) disappear faster than modes of larger energy such as **A**) and **B**). It is expected that the z-gradient coil excited only the mode **B**).

Fig. 2 **A**) describes the exponential decay constant τ, which had a value of 3.805ms in the MIM simulation and mean value of 3.78ms in the experiment, with a confidence interval of 95% in the range 95% of 3.779 ms to 3.781 ms, respectively. The value measured and that predicted by the MIM using Eq.(1) is in good agreement what means that the z-gradient coil excited only the mode **B)**, which proof its existence and at the same time validates the formulation presented in this paper. Fig. 2 **B**) shows the absolute value of the current density induced in the cylinder by the z-coil; the arrows describe the current direction, which has similar profile of that presented in the used z-gradient coil.

Conclusions

This work has presented a simple formulation for the modal analysis of MRI gradient coils using a MIM. The formulation presented was validated by using a z-gradient coil surrounded by a conductive cylinder. We proof the existence of modes and that excited by the z-gradient coil. Understanding the nature of the eddy currents may open the possibility of designing coils with control over the spatiotemporal behavior of the magnetic field in certain region of interest.

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