Proposal of Concept of Theoretical Formula for Equivalent Resistances for Zone-Control Induction Heating System and Theoretical and Numerical Examination

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Abstract —When a workpiece is heated by eddy currents using a zone-control induction heating (ZCIH) system, there exists both inductance and resistance in the induction heating circuit. In order to efficiently control a ZCIH system, the detailed behavior of the inductance and equivalent resistance of each coil, and the mutual inductance and equivalent resistance between the coils, should be clarified beforehand. This paper proposes the concept of self- and mutual-equivalent resistance in the eddy-current circuit, and discusses the theoretical physical meaning and properties of these parameters. We also derive the theoretical formula for these parameters using a simple assumption and then examine their properties.

***Index Terms*—Electromagnetic field, finite element method, induction heating.**

I. INTRODUCTION

A zone-control induction heating (ZCIH) system, which has several coils and where each coil is connected to an independent power supply, has been previously proposed [1-3]. A ZCIH system can heat workpieces uniformly by controlling the amplitude of the current under the condition that the frequency and phase angle of the current in each coil are the same.

When the workpiece is heated by an eddy current caused by the current in the coil, the coil can be treated as both inductance and resistance. The resistance that corresponds to the heat in the workpiece can be treated as an "equivalent resistance," which can be classified as "self-equivalent resistance" and "mutual-equivalent resistance." Self-equivalent resistance appears in a coil when the current is applied to the coil, while mutual-equivalent resistance appears when the current is applied to the other coil. The eddy current in the workpiece can be explained by the equivalent circuit model using equivalent resistances.

In this paper, we propose the concept of using self- and mutual-equivalent resistances for the precise control of a ZCIH system. We derive the theoretical formula of these parameters using simple assumption, and then examine their properties.

II. SELF- AND MUTUAL-EQUIVALENT RESISTANCES

We assume that the inductance in the eddy-current circuit is sufficiently smaller than that in the exciting circuit. From Faraday's law of induction, the interlinkage flux

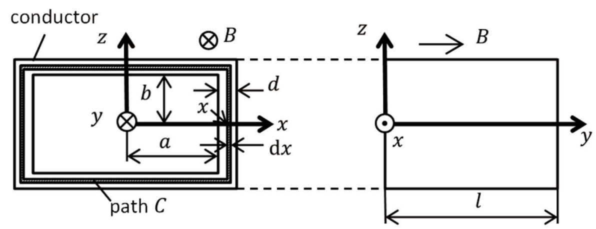


Fig. 1. Brick workpiece of which the inside is vacant.

induced by the eddy current Φ^{e} is proportional to $\mathrm{d}\Phi/\mathrm{d}t$. Therefore, we can express Φ^{e} as follows:

$$\Phi^{\mathrm{e}} = -\kappa\frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{1}$$

where κ is the proportional coefficient. The interlinkage flux in the coil is obtained as follows:

$$\Phi + \Phi^{\mathrm{e}} = Li - \kappa L\frac{\mathrm{d}i}{\mathrm{d}t} \tag{2}$$

Then, the induced voltage is obtained as follows:

$$v(t) = \frac{\mathrm{d}(\Phi + \Phi^{\mathrm{e}})}{\mathrm{d}t} = (\mathrm{j}\omega L + \omega^2\kappa L)\cdot i(t) \tag{3}$$

where ω is the angular frequency of the current and $\mathrm{j} = \sqrt{-1}$. We define $\omega^2\kappa L$ in (3) as R, which is called as a "self-equivalent resistance". When we use the phasor in (3), we obtain the following equation:

$$\dot{V} = (\mathrm{j}\omega L + R)\dot{I} \tag{4}$$

We consider the eddy-current loss of the specimen W_{e}^{*} shown in Fig. 1. By assuming that the opposing field is negligible, we obtain W_{e}^{*} as follows:

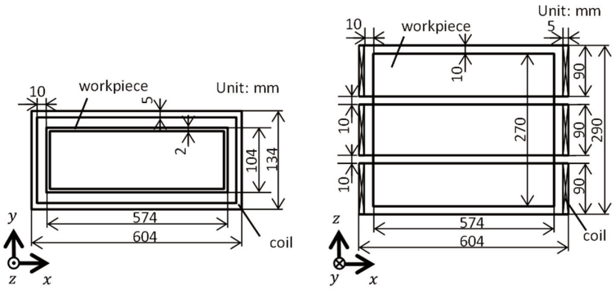
$$\begin{aligned} W_{\mathrm{e}}^{*} &= \sigma l\omega^2 B_{\mathrm{m}}^2\left[\frac{(a+b+2d)^4-(a+b)^4}{64}\right.\\ &\quad -\frac{(a-b)^2}{16}\{(a+b+2d)^2-(a+b)^2\}\\ &\quad \left.+\frac{(a-b)^4}{16}\log\frac{a+b+2d}{a+b}\right] \end{aligned} \tag{5}$$

where B_{m} is the maximum flux density and σ is conductivity. Thus, we have the following relation:

$$B_{\mathrm{m}} = \frac{\Phi_{\mathrm{m}}}{SN} = \frac{LI_{\mathrm{m}}}{SN} = \frac{\sqrt{2}LI}{SN} \tag{6}$$

$$W_{\mathrm{e}}^{*} = RI^2 \tag{7}$$

where Φ_{m} is the maximum value of the flux, S is the area of the coil, N is the number of turns in the coil, and I_{m} and I are the maximum current and the root mean square value of the current in the induced coil, respectively. Finally, we obtain R as follows:



(a) Top view (b) Side view

Fig. 2. Analyzed model of ZCIH.

$$
\begin{aligned}
R \;=\; & \frac{2\sigma l \omega^2 L^2}{S^2 N^2}\Bigg[\frac{(a+b+2d)^4-(a+b)^4}{64} \\
& -\frac{(a-b)^2}{16}\{(a+b+2d)^2-(a+b)^2\} \\
& +\frac{(a-b)^4}{16}\log\frac{a+b+2d}{a+b}\Bigg]
\end{aligned} \tag{8}
$$

When there are many coils, we obtain the following equation in the same manner:

$$
\begin{aligned}
v \;&= \Phi_m + \Phi_m^{\mathrm{e}} + \sum_{n;m\neq n}(\Phi_{mn}+\Phi_{mn}^{\mathrm{e}}) \\
&= L_m i_m - \kappa_m L_m \frac{\mathrm{d}i_m}{\mathrm{d}t} \\
&\quad + \sum_{n;m\neq n}\left(M_{mn} i_n - \kappa_{mn} M_{mn}\frac{\mathrm{d}i_n}{\mathrm{d}t}\right)
\end{aligned} \tag{9}
$$

where, i_m is the current in coil m, Φ_m is the interlinkage flux in coil m induced by itself, Φ_{mn} is the interlinkage flux in coil m induced by coil n, and κ_m and κ_{mn} are the proportional coefficients corresponding to coil m itself and coils m and n, respectively.

When we define $\omega^2\kappa_m M_m$ as R_m (which is called "self-equivalent resistance"), and $\omega^2\kappa_{mn} M_{mn}$ as R_{mn} (which is called "mutual-equivalent resistance"), we obtain the following equation:

$$
\dot{V}_m = (\mathrm{j}\omega L_m + R_m)\dot{I}_m + \sum_{n;m\neq n}(\mathrm{j}\omega M_{mn} + R_{mn})\dot{I}_n \tag{10}
$$

If we substitute $\mathrm{j}\omega L_m$ for $(\mathrm{j}\omega L_m + R_m)$ and $\mathrm{j}\omega M_{mn}$ for $(\mathrm{j}\omega M_{mn} + R_{mn})$ in (10), respectively, the equation corresponds to the well-known equation of a transformer.

III. Equivalent Resistance Coupling Coefficient

The inductance coupling coefficient between coils m and n, k_{mn} is expressed as follows:

$$
k_{mn} = \frac{M_{mn}}{\sqrt{L_m L_n}} \tag{11}
$$

Similarly, we define the equivalent resistance coupling coefficient between coils m and n, k_{mn}^{R} as follows:

$$
k_{mn}^{\mathrm{R}} = \frac{R_{mn}}{\sqrt{R_m R_n}} \tag{12}
$$

Using (10) and (12), we obtain k_{mn}^{R} as follows:

$$
k_{mn}^{\mathrm{R}} = \frac{R_{mn} I^2}{\sqrt{R_m I^2 \cdot R_n I^2}} = \frac{W_{m,n}^{\mathrm{Total}} - W_m - W_n}{2\sqrt{W_m W_n}} \tag{13}
$$

where $W_{m,n}^{\mathrm{Total}}$ is the eddy-current loss when the current I paths through coils m and n.

If the workpiece is sufficiently thin and close to the coil, and the relative magnetic permeability of the workpiece approaches unity, we obtain the following equation using (13):

$$
k_{12}^{\mathrm{R}} = \frac{2k_{12} + k_{12}k_{13}}{\sqrt{(k_{12}^2 + k_{13}^2 + 1)(2k_{12}^2 + 1)}} \tag{14}
$$

$$
k_{13}^{\mathrm{R}} = \frac{k_{12}^2 + 2k_{13}}{k_{12}^2 + k_{13}^2 + 1} \tag{15}
$$

TABLE I

SELF-EQUIVALENT RESISTANCE

	R_1 [Ω]	R_2 [Ω]
FEM	1.08	1.10
Estimate	1.13	0.95

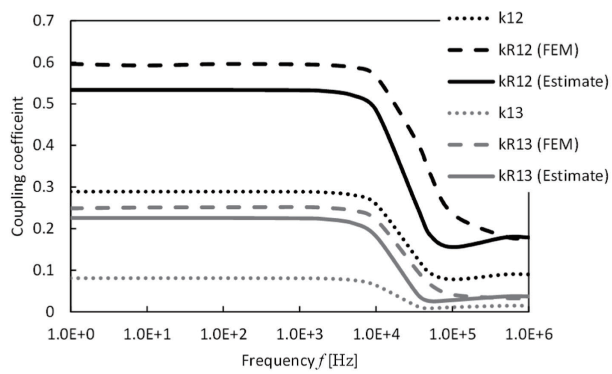


Fig. 3. Frequency characteristics of inductance coupling coefficient and equivalent resistance coupling coefficient.

IV. Results and Discussion

Table I shows R_1 and R_2 using FEM and estimated using (8) when the resistivity is 1200 μΩ·cm and the frequency of the current is 35 kHz. The estimated R_1 and R_2 are almost the same as those calculated by FEM.

We calculate k_{mn}^{R} using FEM, and estimate k_{mn}^{R} using (14) and (15). In (14) and (15), we use k_{mn} calculated by FEM. Figure 3 shows the frequency characteristics of k_{mn}, k_{mn}^{R} (FEM), and k_{mn}^{R} (estimate). We obtain a fairly good agreement between k_{mn}^{R} (FEM) and k_{mn}^{R} (estimate).

In this paper, we derive the theoretical formula for R_m and R_{mn}: We can estimate R_m using (8), and k_{mn}^{R}, which is necessary to calculate R_{mn} using (14) and (15). The details of the equations' derivation will be shown in the full paper.

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