Dual Finite Integral Technique in Ion Flow Field Calculation

Yongsheng Xu, Jinliang He, *Fellow IEEE* and Bo Zhang
Department of Electrical Engineering, Tsinghua University, Beijing, 100084, China
Emexplorer@163.com

***Abstract*—The calculation of ion flow field in the electromagnetic environment of high voltage direct current (HVDC) transmission lines is necessary. The calculation involves the electrostatic problems and ion movement problems. In this paper, Finite Integral Technique (FIT) is used in the calculation of ion flow field of HVDC transmission lines, and the dual formulations of FIT on dual meshes are proposed. By the numerical test, the method is proved to be valid. And the quick calculation is realized.**

***Index Terms*—**Electric field, finite integral technique (FIT), ions, high voltage directs current (HVDC) transmission lines.

I. Introduction

The corona phenomenon of the high voltage direct current (HVDC) transmission lines will generate positive or negative ions. In the electric field, these ions flow in the certain directions called ion flow field. The ion flow field will enhance the original electric field on the ground level. Thus the calculation of the ion flow field becomes an important problem to the electromagnetic environment of HVDC transmission lines.

To solve this problem, upwind finite element method and finite volume method was used [1]. Due to the very large domain to be calculated and very small size of the transmission lines, these methods have some problems.

Finite integral technique (FIT) first induced by Weiland [2], is now widely used in wave propagation problems. The characteristics of FIT are faster but less accurate than finite element method (FEM). Ren has proved that the upper and lower energy boundaries of dual FIT on dual meshes are exist both in the electrostatic problems and eddy current problems as well as the FEM [3][4].Thus using FIT on the rough dual meshes can get fast and accurate results. It is appropriate to use dual FIT method to calculate the electric field of transmission lines. In this paper, we use dual formulations of FIT in the calculations of ion flow field to get fast and accurate result.

II. Basic Formulations

From the point of view, bipolar design above ground is more of interest. In this paper, the bipolar ion flow field is calculated. Following assumptions are needed:

1. The thickness of the corona ionization layer is negligible.
2. Ionic mobility is constant, and diffusion of ions is negligible.
3. The effect of wind, humidity, aerosols, etc. is neglected.
4. The magnitudes of electric field strength at coronating conductor surfaces remain constant at their onset value.

Governing formulations:

$$divD = \rho \tag{1}$$

$$J = k\rho E \tag{2}$$

$$divJ = 0 \tag{3}$$

$$D = \varepsilon E \tag{4}$$

Where E, D, J, ρ are electric field, electric flux density, electric current density and charge density; K is ionic mobility (It is different for the conditions of positive ions and negative ions).ε is the permeability of free space. (1) is Poisson's equation, (2) defines the relationship between the ion current density and electric field, (3) is continuity equation of ion current density, and (4) is the constitutive law.

III. Dual Formulations of FIT

Most methods to calculate the ion flow fields may be divided into two parts: calculations of the electric fields and the ion flow fields.

A. Definitions of FIT

Like most differential numerical methods, FIT works on the mesh to get numerical solutions. Working variables are associated with the points (0-form), the edges (1-form), the facets (2-form), and volumes (3-form) of the mesh. In this problem working variables can be defined as:

0-form $\bar{\varphi}$

1-form $\bar{e} = E \cdot \Delta l$

2-form $\bar{d} = D \cdot \Delta s$, $\bar{j} = J \cdot \Delta s$

3-form $\bar{\rho} = \rho \cdot \Delta v$

Where Δl , Δs , Δv , are small length, area, volume of the mesh. Working variables $\bar{\varphi}$, $\bar{e}$, $\bar{d}$, $\bar{j}$, $\bar{\rho}$ are the discrete forms of electric potential , electric field, electric flux density, electric current density, and charge density on the mesh. From the physical point of view $\bar{\varphi}$, $\bar{e}$, $\bar{d}$, $\bar{j}$, $\bar{\rho}$ are also electric potential, voltage, electric flux, electric current and charges.

The discrete form of the constitutive coefficient is called Hodge operator:

$$\bar{d} = M_{\varepsilon}\bar{e} \tag{5}$$

$$\bar{j} = M_{\sigma}{}'\bar{e} \tag{6}$$

(5) refers to the capacitance matrix of the meshes, and (6) refers to the conductance matrix of the mesh. In steady state they are constant.

In the case of orthogonal mesh, M_{ε} and $M_{\sigma}{}'$ are diagonal matrix. The matrix elements can be easily computed:

$$m_{\varepsilon ii} = \varepsilon \frac{\Delta s}{\Delta l} \tag{7}$$

$$m_{\sigma ii}{}' = k\rho \frac{\Delta s}{\Delta l} \tag{8}$$

The conductance matrix is related to the charge density, we can change (8) to:

$$m_{\sigma ii}{}' = \bar{\rho} m_{\sigma ii} \tag{9}$$

And:

$$m_{\sigma ii} = k \frac{\Delta s}{\Delta v \Delta l} \tag{10}$$

The main difference between FIT and FEM is the discrete form of the constitutive coefficient. The FIT is simpler and less accurate than FEM.

B. *Functions of the FIT*

As the definition of the variables above, discrete forms of operators of curl, divergence, and gradient are C D and G. They are depended by the geometry of the mesh, and the forms are like [-1 1 0 ⋯, 1, ⋯].The discrete form of Poincare lemma shows that: CG=0 and DC=0.

$$DM_{\varepsilon} G\bar{\varphi} = \bar{\rho} \tag{11}$$

$$\bar{j} = \bar{\rho} M_{\sigma} G \bar{\varphi} \tag{12}$$

$$D\bar{j} = 0 \tag{13}$$

Salved (10) and (11) can get:

$$D\bar{\rho} M_{\sigma} G\bar{\varphi} = 0 \tag{14}$$

C. *Dual formulations on dual meshes*

When the variables are associated with the meshes (in three dimensions formed by tetrahedron or hexahedron elements), we can map the nodes, edges, facets, and volumes of the mesh by Hodge transforms to the volumes, facets, edges, and nodes of the dual mesh.

Delaunay triangle and Voronoi diagram are a kind of dual meshes. By Ren's dual mesh complex [3], allocating φ to the points of primal mesh, e to the edges of the primal mesh, d and j to the facets of dual mesh and ρ to the volume of dual mesh just like fig 1.we get the primal set of equations.

$$\begin{tikzcd} \bar{\varphi}_p \arrow[r, "G"] & \bar{e}_p \arrow[d, leftrightarrow] \\ \bar{\rho}_d & \bar{d}_d \arrow[l, "G^T"'] \end{tikzcd} \qquad \begin{tikzcd} \bar{\varphi}_p \arrow[r, "G"] & \bar{e}_p \arrow[d, leftrightarrow] \\ 0 & \bar{j}_d \arrow[l, "G^T"'] \end{tikzcd} .$$

Fig.1 Formulations on the primal mesh

$$G^T M_{\varepsilon}^{p} G\bar{\varphi}_p = \bar{\rho}_d \tag{15}$$

$$G^T \bar{\rho}_d M_{\sigma}^{p} G\bar{\varphi}_p = 0 \tag{16}$$

Allocating φ to the points of dual mesh, e to the edges of the dual mesh, d and j to the facets of primal mesh and ρ to the volume of primal mesh, we get the dual set of equations.

$$DM_{\varepsilon}^{d} D^T \bar{\varphi}_d = \bar{\rho}_p \tag{17}$$

$$D\bar{\rho}_p M_{\sigma}^{d} D^T \bar{\varphi}_d = 0 \tag{18}$$

These functions are circuit network style: (15) and (17) are capacitance network; (16) and (18) are Kirchhoff's current law on the meshes.

D. *Numerical results*

A 2-D example of ±400kv bipolar dc transmission lines is calculated. The parameters and measured data are presented by [5].

The primal and dual mesh is all rectangles. The process of calculation is:

Step 1: calculating the space charge free electric field.

Step 2: initializing the charge density on the conductor surface.

Step 3: calculating the space charge distribution by (16) and (18) on dual meshes separately.

Step 4: calculating the electric field by (15) and (17) separately, and refreshing the electric field.

Averaging the two results of dual meshes, if its error limit to the corona onset electric field on the conductor surface is small enough, calculation is completed, else refreshing the charge density and go to step 3.

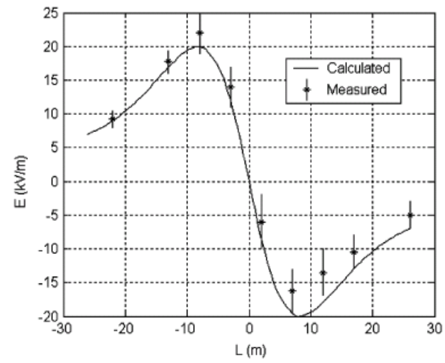

Fig.2 calculated and measured date of electric field on the ground level

Because using dual meshes, calculation can be put on the rough mesh to be very quick. From the result, we can see that the method is valid. Due to the circuit network style functions of the FIT, it is possible to get equivalent circuit network of ion flow field problem.

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