# A Shimming Scheme for Active Shielding

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***Abstract*—A careful selection of a basis for the field representation is a useful tool in the active shielding design. When the domains of interest are characterized by simple geometry in orthogonal coordinate systems, and linear homogeneous materials are involved, the product solution in eigenfunction series is very effective thanks to the availability of analytical solutions and to the convergence properties of the sum. Taking advantage of analytical expressions of single eigenfunction series terms, the paper shows how to design in advance suitable elementary source, well suited to perform the shielding of fields in a direct and prompt way.**

***Index Terms*—Magnetic Shielding. Eigenfunctions, Shielding sources.**

## I. Introduction

The shielding of electromagnetic fields is critical task for a wide class of important applications, including biomedical engineering, field detection in research activities, critical apparatus protection [1-5]. The classical "passive approach" assigns the mission to counteract undesired fields in a volume of interest to suitably designed materials. The set of shielding materials includes the magnetic ones, well suited to react to magnetic disturbing field, and the conductive materials, able to counteract the static electric fields by grounding, and dynamic fields by means of induced currents.

An effective alternative is the "active shielding" (AS) approach, based on the use of suitably designed shielding sources. The AS is well suited especially when the shape and the frequency content of external field is not known in advance and/or it changes time to time. In fact, in this cases, the AS system can be designed to use a number of independently driven sources (charges, currents), whose actual values can be chosen according to the characteristics of the particular field, to provide the best shielding effect.

In order to facilitate the mathematical description of the problem, a suitable functional basis for the field representation can be chosen. Then, both the external and the shielding fields are fully characterized by just the set of basis coefficients.

In addition, a number of "Basis Sources" (BS) can be designed, each specialized to generate a single unitary element of the basis. The identification of the coefficients of the external field in the chosen basis directly provides the "amplitude" of BS needed for the shielding action. This approach is frequently referred to as "shimming".

Aim of this paper is to propose the adoption of BS derived from the analytical solutions of suitably formulated Laplace equations. The approach proposed in the following can be used for both electrostatic and magnetostatic linear problems. It can also be used in case of dynamical problem, and the impact of the possible induced currents must be included; fortunately, thanks to the linearity, also such effect can be spanned in the same representation basis, then saving the general properties of the methodology.

The proposed approach is based on the possibility, in a number of limited domains, characterized in a specific orthogonal coordinate system by a highly symmetric geometry, of satisfying the Laplace equation by series of products of well known *eigenfunctions*; in addition, the series posses a number of strong and useful properties, such as the convergence of the summation, and the evaluability of the truncation error in closed form.

Of course, the approach requires the linearity because it is based on the superposition theorem. However, it can properly be used also in procedures able to face with non linear problems by solving a number of iterative linear steps.

In this digest, for the sake of exemplification, the mathematical formulation is summarized in the particular case of cylindrical volumes; the basis for the conceptual design of the BS, designed to generate, within an assigned approximation, each a single harmonic are provided.

In the full paper the AS problem is formulated in a generic reference frame, including the gathering of information about the external field, and both magnetostatic and electrostatic cases are considered.

## II. Mathematical Formulation

In a source-free, simply connected domain $\Omega$, filled with a homogeneous isotropic material, the mathematical model for linear magnetostatic problems can be written in terms of a scalar potential $\psi(\mathbf{r})$:

$$\begin{cases} \nabla^2\psi(\mathbf{r}) = 0 \\ \mathbf{H}(\mathbf{r}) = -grad\,\psi(\mathbf{r}) & \mathbf{r}\in\Omega \\ \mathbf{B}(\mathbf{r}) = \mu\mathbf{H}(\mathbf{r}) \end{cases} \quad (1)$$

with suitable boundary conditions added on the domain boundary $\partial\Omega$.

### *A. Eigenfunction Expansion*

In the particular case of a cylindrical box, described in a cylindrical coordinate system (see Fig.1), and assuming Dirichlet boundary conditions, the representation basis include three series, one for each basis (top and bottom), and one for the lateral surface. Each series provides the solution related to a single "active" face, with homogeneous Dirichlet conditions on the others. For example, the terms related to the bottom side, at z=0, is given by [6]:

$$\begin{cases} \psi(\rho,\theta,z)=\sum_{n}\sum_{m} A_{nm}P_{nm}(\rho)\Phi_n(\varphi)Z_m(z) \\ P_{nm}(\rho)=J_n(k_m\rho) \\ \Phi_n(\varphi)=\cos(n\varphi)\ \text{or} \sin(n\varphi) \\ Z_m(z)=\cosh(|k_m|z+\alpha) \end{cases} \qquad (2)$$

where $J_n$ is the n-th order Bessel function. To force the homogeneous boundary condition on the lateral surface, $k_m a$ must be a zero of the Bessel function, and $\alpha$ is chosen to force zero at z = $h$.

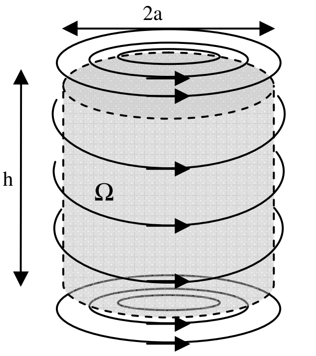


Fig. 1. Cylindrical domain and current system for the *(0, 1)* harmonic related to the top face at z=0.

In the following, a single term of the representation basis, satisfying Dirichlet condition on one of the sides and characterized by indexes m and n, will be referred to as a "*(m, n)* harmonic" for the side. For any external field, the accuracy in the representation of its potential depends only on the number of *(m, n)* harmonics considered in the expansion.

### *B. Design of BS*

For each harmonic of the expansion, a BS can be easily defined in the free space assuming regularity condition at infinite. For example, the BS for the *(m,n)* harmonic related to the top face is the surface current distribution required, in each point of the domain boundary ∂Ω, to generate the *(m,n)* harmonic in the potential distribution within the domain Ω.

Of course, the mathematical model behind the "design" of each BS falls in the class of Freedholm equations, where the weight function is given by the Green function in free space projecting the unknown surface current density onto the potential function ψ. Note that the method is applicable to quite general classes of similar problems, eventually including double-layers surface sources.

In order to take advantage of the chosen expansion, each BS can be in turn projected on the representation basis; in this way each BS is characterized by the simple set of amplitudes of the basis harmonics. Note that, due to the coupling among the sides, in principle all the harmonics for all the sides are not vanishing, but, on the other hand, thanks to the known convergence properties of the expansion, the number of harmonics to be actually considered to guarantee the requested accuracy in the full domain can be easily chosen.

Fortunately the BS design must be performed just once for each domain Ω to be shielded, preliminary to the actual shielding action. When an external field must be shielded, the required action is simply the evaluation of the amplitudes of each BS required to shield the particular field (e.g. the current amplitudes, but not their distribution): the approach is therefore well suited for real time applications.

In the case of Neumann conditions on some side, the BS would include suitable more complex sources, and the equations will include the related Green Function; however, the mathematical modeling keeps the same structure [7].

## III. Example of Application

In order to demonstrate the main steps of the approach, the procedure to generate *(0, 1)* harmonic for a cylinder with radius 1m and height 4m is sketched. The coil system considered here is a system of 200 concentric circular coils, whose radii are evenly distributed from the center to 1 m at z=0 and z=4m and a system of 200 coils with radius 1m equally distributed along the lateral surface of the cylinder. The field associated to the considered potential harmonic is sampled inside the cylinder and this quantity is used to determine the optimal currents that fed each coils. The relative error between the reconstructed field and the harmonic field is below 30dB in all points of the domain Ω. In fig. 2 the obtained current distribution on the disc at z=0 is plotted.

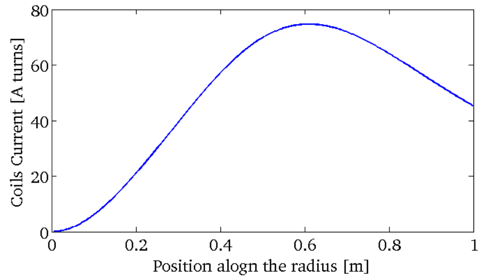


Fig. 2. Currents in coils system on bottom face generating the *(0, 1)* potential harmonic associated to the same face.

## IV. Outlook

A design procedure to generate source distributions able to shield volumes from undesired external fields has been sketched, and demonstrated with a simple example. In the full paper, a more general description of the design procedure will be presented, and examples of applications will be given.

## V. Acknowledgements


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