A novel tool for breakdown probability predictions on multi-electrode multi-voltage systems

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***Abstract*—An innovative approach for the voltage breakdown prediction in high voltage systems, insulated by large vacuum gaps, has been recently formulated and experimentally benchmarked, which is based on the modelling of the whole system in terms of breakdown probability by Weibull's distribution, provided an infinitesimal breakdown probability can be associated to each particle trajectory between different electrodes. In this work an analytical solution is proposed to calculate the particle trajectories over a tetrahedral mesh, by means of complementary formulations of the electrostatic field laws. This tool is benchmarked on a reference configuration and applied to estimate the breakdown probability for a very complex multi-electrode multi-voltage system.**

***Index Terms*—Electrostatics, Ion beams, Finite element methods**

I. Introduction

Although the high vacuum is used extensively for the insulation of high voltage devices in many research and industrial areas, at present no consolidated design criteria are available in literature to predict the breakdown probability of a multi–electrode multi–voltage system insulated by large vacuum gaps. Nevertheless, an innovative approach for the voltage breakdown prediction of such complex systems has been recently formulated [1] and benchmarked with experimental results, which is based on the correlation between the the clump mechanism [2], [3] and a statistical approach applied to the breakdown probability associated to the trajectory of electrically charged micro-particles leaving one electrode and clashing to the electrode with opposite polarity with sufficient energy to get a vaporization.

Here an analytical solution is proposed to calculate the particle trajectories over a tetrahedral mesh by means of complementary formulations for electrostatics. This tool has been benchmarked on a reference configuration (sphere/plane) problem and applied to estimate the breakdown probability for a very complex multi-electrode multi-voltage system.

II. The micro-particles probabilistic model

An electrostatic system is considered, which is composed by a number of electrodes polarized at different voltages (V_i). The breakdown probability associated to the macroscopic area A_i of the i-th electrode can be expressed as

$$p_i = n_i A_i \tag{1}$$

where n_i denotes the number of micro-particles per surface unit which can be detached by electrostatic forces and impinge an other electrode with sufficient energy to get a vaporization.

Using the same approach followed by the Failure Analysis Theory [4], the voltage holding probability R_H has the following expression

$$R_H = \prod_{i=1}^{M} (1 - p_i) \tag{2}$$

being M the overall number of subdivisions of the different electrodes from which the particles can be detached. Then, the breakdown probability for the whole system is

$$P = 1 - R_H \tag{3}$$

The crucial point is to identify the relationship between n_i and the physical mechanism underlying the clump induced breakdown. According to the Slivkov-Cranberg theory, the nonzero probability breakdown will occur when the following condition is fulfilled for a particle travelling from a point P on the cathode to a point Q on the anode

$$W = \Delta V \, E(P) \, E(Q)^{2/3} > W_S \tag{4}$$

where E denotes the norm of the electric field and ΔV the voltage between the two electrodes.

The fundamental assumption is that the number of micro particles that potentially can produce a breakdown is a monotonic function of the W parameter; this functional dependence can be expressed by the Weibull's distribution

$$n(W) = \left(\frac{W}{W_0} \right)^m \tag{5}$$

where W_0 and m are a scale parameter and a shape parameter, respectively.

III. Electrostatic complementary formulations

The proposed tool is based on the analytical calculation of the particle trajectories by means of complementary formulations of the electrostatic field laws, which have been implemented into a numerical code (CAFE). A dedicated tool is used to compute the particle trajectories neglecting the collisions. The CAFE code uses the standard FEM formulation $\mathbf{V}$ based on the electric scalar potential together with

an original formulation $\mathbf{P}$ that employs the electric vector potential [5]. The advantage of using two formulations arising from complementary potentials is that the constitutive error is minimized [6] and an accurate solution may be obtained as the mean between the ones produced by the pair of complementary formulations. Nonetheless, complementarity requires a complicated topological pre-processing [5] and that is why it is not widely used in practice.

The computational domain is covered by a tetrahedral mesh whose incidences are encoded in the *cell complex* $\mathcal{K}$ represented by the standard incidence matrices $\mathbf{G}$, $\mathbf{C}$ and $\mathbf{D}$. A dual barycentric complex $\tilde{\mathcal{K}}$ is obtained from $\mathcal{K}$ by using the *barycentric subdivision*. The matrices $\tilde{\mathbf{G}} = \mathbf{D}^T$, $\tilde{\mathbf{C}} = \mathbf{C}^T$ and $\tilde{\mathbf{D}} = -\mathbf{G}^T$ represent the incidence matrices of $\tilde{\mathcal{K}}$. In order to formulate the problem by using the vector potential formulation, an array of voltages $\tilde{\mathbf{U}}$ on dual edges $\tilde{e}$, an array of electric fluxes $\mathbf{\Phi}$ on faces f and an electric vector potential $\mathbf{P}$ on edges e are introduced. The vector potential, that in absence of source charges is defined through

$$\mathbf{\Psi} = \mathbf{C}\mathbf{P} + \sum_{i=1}^{N} Q_g^i \mathbf{\Pi}^i, \tag{6}$$

verifies the discrete Gauss' law $\mathbf{D}\mathbf{\Psi} = \mathbf{0}$, where $\{\mathbf{\Pi}^i\}_{i=1}^N$ is a set of *thick links*, $N+1$ is the number of electrodes and $\{Q_g^i\}_{i=1}^N$ is the corresponding set of independent induced electric charges, one for each electrode but one that is chosen as a reference electrode. Concerning boundary conditions, $\mathbf{P}$ is set to zero for all edges belonging the portion of $\partial\mathcal{K}$ subject to homogeneous Neumann boundary conditions. The constitutive matrix $\mathbf{H}$ relates the flux $\mathbf{\Psi}$ to voltage $\tilde{\mathbf{U}}$ with $\tilde{\mathbf{U}} = \mathbf{H}\mathbf{\Psi}$. Concerning the construction of $\mathbf{H}$, one can use the standard mass matrix of Finite Elements produced with face basis functions. Another efficient solution is to use the piecewise-uniform face vector basis functions defined in [7].

By combining the Faraday's discrete law $\mathbf{C}^T\tilde{\mathbf{U}} = \mathbf{0}$ with (6) and the constitutive law, we get

$$\mathbf{C}^T\mathbf{H}\mathbf{C}\mathbf{P} + \sum_{j=1}^{N} \left(\mathbf{C}^T\mathbf{H}\mathbf{\Pi}^j\right) Q_g^j = \mathbf{0}. \tag{7}$$

Also a set of *non-local Faraday's laws* [5] has to be written on each thick link as $\mathbf{\Pi}^{iT}\tilde{\mathbf{U}} = U_g^i$, $i \in \{1, \ldots, N\}$, where U_g^i is the desired voltage between the reference electrode and the ith electrode. The equation is expanded as

$$\left(\mathbf{\Pi}^{iT}\mathbf{H}\mathbf{C}\right)\mathbf{P} + \sum_{j=1}^{N} \left(\mathbf{\Pi}^{iT}\mathbf{H}\mathbf{\Pi}^j\right) Q_g^j = U_g^i. \tag{8}$$

The final algebraic linear system (sparse, symmetric and singular) is iteratively solved without applying any gauge.

The particle trajectories are computed analytically inside each tetrahedra (the electric field is uniform) and the complete trajectory can be obtained by moving between adjacent cells on the basis of a very efficient algorithm, avoiding the standard integration of the particle motion equation by a common Runge-Kutta scheme.

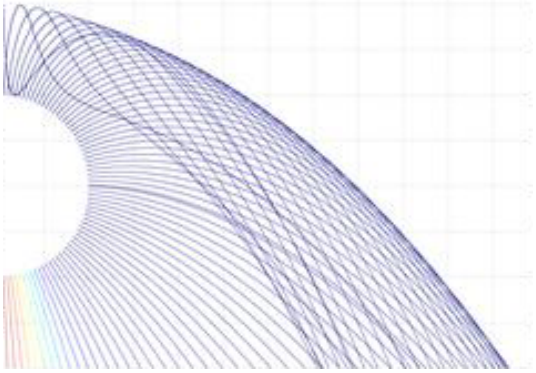

Figure 1: Sphere – plane configuration: particle trajectories.

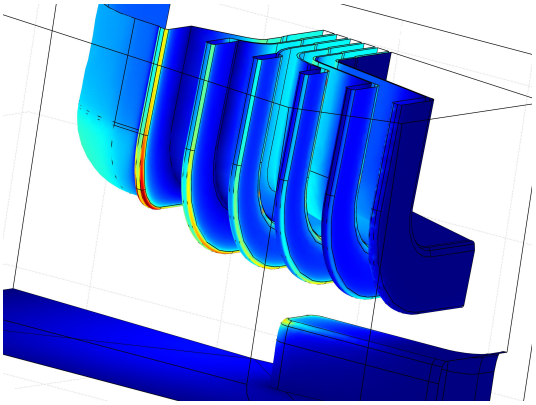

Figure 2: Multi–electrode system: electric field norm.

IV. Numerical results

The proposed approach has been benchmarked on a reference configuration (sphere/plane) problem, see Figure 1, and applied to estimate the breakdown probability for a very complex multi-electrode multi-voltage system, namely the beam source of the $40\,MW$ Neutral Beam Injector (NBI), which is under design in Padova (I) in the framework of the ITER project). In Figure 2 the electric field norm on the Beam source's electrodes is shown. In the full paper more results will be presented and discussed.

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