Time Domain Analysis of Transient Currents in Conductors Involving Non-Homogeneous Media

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***Abstract* — The behavior of problems characterized by time varying electromagnetic fields in conductor media can be described in terms of the current density vector by an integral equation. For complex structures involving non-homogeneous media in multi-conductors systems, for example, numerical procedures in time domain are useful for the solution of transient current distribution in some predefined points of the geometry being considered at each chosen time step.**

The application of Moment Method in association with a finite difference approximation of time derivatives results in a linear system of equations described by matrix-form equations. The paper proposes a novel methodology based on the static Finite Element Method for determining the coefficients of the problem main matrix.

Examples of calculated transient current distributions in some multi-conductors systems configurations are presented as an application for the prediction of crosstalk.

***Index Terms*—Electromagnetic compatibility and interference, mathematics.**

I. Introduction

Many problems concerning the analysis of current distribution in conductive media refer basically to skin and proximity effects, and eddy currents. Those phenomena are responsible for changing the profile of current distribution and in general can lead to increased losses and heating of the conductive material [1].

Time varying electromagnetic fields in conductive media can be analyzed in terms of the current density vector and for complex structures, numerical procedures are employed either in frequency domain or in time domain and, among them, the Moment Method (MoM) can be applied [2].

In the case of problems involving transient phenomena, a frequency domain approach is possible and the solution is obtained successively for different frequencies that represent the spectrum of the excitation function. The limitation of this approach is related to Gibbs phenomenon which leads to difficulties in correctly describing a transient excitation, especially in the case of waveforms with small rise and fall times, as a square wave, for example. A time domain analysis is indicated in this case to obtain accurate solutions, and then the Moment Method is employed in conjunction with a finite difference approximation of the time derivative, leading to matrix-form equations [3].

When dealing with non-homogeneous regions, the elements of the system main matrix can be obtained by means of a static Finite Element Method (FEM) code, such as the version available in the MATLAB PDE toolbox.

The paper shows an application example solved by the developed procedure that concerns transient current distribution in a multi-conductor system formed by non-magnetic bodies in some different configurations [4].

As the proposed method works in a time step by step scheme, it is interesting, for increasing computational efficiency, to reduce the size of the involved matrices. This can be done by application of the Discrete Wavelet Transform [5], not presented in this paper.

II. EM Fields in Conductors

The use of auxiliary vector potentials as aids in obtaining solutions for the electric and magnetic fields is a common practice in the analysis of electromagnetic boundary-value problems. Considering a linear, homogeneous and isotropic medium, and by the definition of the magnetic vector potential $\vec{A}(t)$, the current density $\vec{J}(t)$ in a conductor region with conductivity σ, excited by a given electric field $\vec{E}_O(t)$, can be calculated by

$$\vec{J}(t) = -\sigma \cdot \frac{\partial \vec{A}(t)}{\partial t} + \vec{J}_o(t)\,, \tag{1}$$

where $\vec{J}_o(t) = \sigma \cdot \vec{E}_o(t)$.

Assuming that there are no variations in the electric field in the $\vec{a}_z$ direction, $\vec{J}$ depends only on the (x,y) position in Cartesian coordinates and the problem becomes bi-dimensional with $\vec{A} = A_z \vec{a}_z$.

In the case of conductive media, where the displacement current can be neglected, the magnetic vector potential, imposing $\vec{\nabla} \cdot \vec{A} = 0$, relates to the current density by

$$\nabla^2 \vec{A} = -\mu\, \vec{J}\ . \tag{2}$$

Equation (2) shows that $\vec{A}(t)$ is defined by an integral of the current distribution, so that (1) takes the form of an integral equation on $\vec{J}(t)$.

In the case of a very long non-magnetic conductor, the relationship between the vector potential and the current density in some arbitrary cross section of the conductor can be obtained numerically by subdividing it into filamentary elements aiming to the application of the Moment Method. Thus, applying the Moment Method and substituting the time derivative by a finite difference approximation, the integral equation that describes the problem is substituted by a matrix representation:

$$[J]^{(n+1)} = [I+C]^{-1} \cdot [C] \cdot [J]^{(n)} + [I+C]^{-1} \cdot [J_o]^{(n+1)}. \qquad (3)$$

The upper index n is associated to $t = n \cdot \Delta t$ (Δt is the computational time step), $[I]$ is the identity matrix and $[C]$ is a matrix whose general element c_{ki} is a coefficient that gives the contribution of element i on element k of the spatial subdivision.

In situations involving the presence of magnetic materials where $\mu \neq \mu_o$, a static FEM code can be used in order to obtain the magnetic vector potential A_z in some pre-determined points of the conductors (considering filamentary currents imposed). The numerical relationship obtained between the potential at a point i(xi,yi) and the imposed current element at a point j(xj,yj) permits, in this case, to determine matrix $[C]$.

This procedure was validated considering various situations with known solution and proved to be an efficient tool for analyzing problems of practical interest.

III. Application

Fig. 1 shows a cross section of a multi-conductor system configured with 4 pairs of source conductors (a, b, c and d, defining four circuits) and a victim conductor. Conductor victim is supposed to have their extremities short circuited. The pairs of conductors a, b, c and d are subjected to imposed fields $E_o(t)$ so that the associated transient currents J_a, J_b, J_c and J_d are as indicated in Fig. 1 considering two situations. The system was configured with 9 filamentary elements leading to a matrix $[C]$ with 9x9 coefficients. The time step adopted was $\Delta t = 1\ \mu s$ for the procedure to be stable.

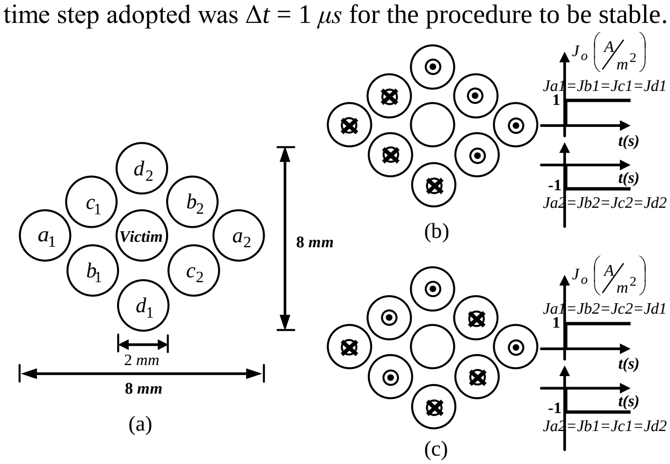

Fig. 1. (a) geometric characteristics of the problem; (b) excitation distribution of the source conductors – configuration 1; (c) excitation distribution of the source conductors – configuration 2.

Fig. 2 shows the calculated waveforms of current densities specifically for conductor a_1. The conductors parameters used in this simulation were $\sigma = 1.0 \cdot 10^5$ (S/m) and $\mu_r = 100$ for both configurations 1 and 2. For a coherent comparison analysis the same problem was solved considering all the conductors with $\sigma = 5.8 \cdot 10^2$ (S/m) and $\mu_r = 100$ only for configuration 1.

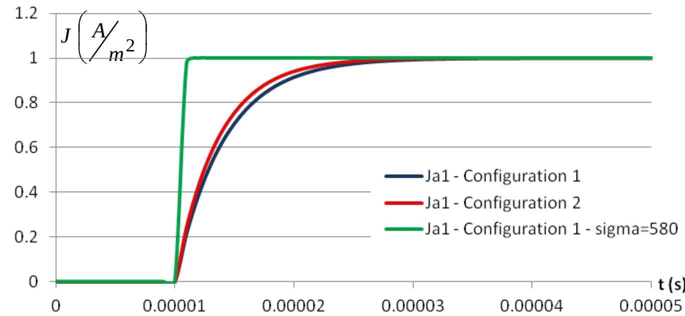

Fig. 2. Simulation results for the current densities at conductor a_1.

It can be noticed that the waveforms of current densities of source conductors are strongly influenced by the increasing of their conductivities. It is caused due to the greater interaction between conductors in this case.

Fig. 3 shows some results obtained for the victim conductor considering the situations previously analyzed.

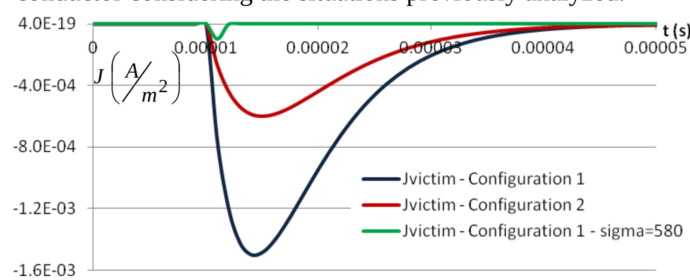

Fig. 3. Simulation results for the current densities at victim conductor.

The results show a more pronounced reduction of the victim current in the case of lower conductivity. As expected, it can be observed comparing configurations 1 and 2, that there is a lower interference in the victim conductor when the way of some source currents are inverted.

IV. Conclusions

The paper has presented an efficient numerical procedure for time domain analysis of transient currents in the case of non-homogeneous media.

Some particular configurations of multi-conductors were analyzed and the results for the current densities have shown the effect of the conductivity and the way source currents flow on both the victim and source conductors.

V. References

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