Surface integral equations for electromagnetic testing: the low-frequency and high-contrast case

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***Abstract*—This study concerns boundary element methods applied to electromagnetic testing, for a wide range of frequencies and conductivities. The eddy currents approximation cannot handle all configurations, while the common Maxwell formulation suffers from numerical instabilities at low frequency or in presence of highly contrasted media. We draw on studies that overcome these problems for dielectric configurations to treat conductive bodies, and show how to link them to eddy current formulations under suitable assumptions. This is intended as a first step towards a generic formulation that can be modified in each sub-domain according to the corresponding medium.**

***Index Terms*—Eddy currents, low frequency, Maxwell equations, surface integral equations.**

I. Introduction

This study is motivated by the need to efficiently simulate complex configurations of electromagnetic non-destructive testing (ENDT). Complexity may lie in the geometry and also in the cohabitation of different models: an eddy current model (EC) in conductive parts, a magneto-static model in non-conductive permeable parts (e.g. ferrite cores) and Maxwell equations in parts where the displacement current cannot be neglected [1, Ch. 8] or in weakly conductive parts tested at higher frequencies (e.g. composite media). Here we restrict the study to isotropic and piecewise homogeneous linear media. The boundary element method (BEM) allows intuitive domain decomposition. Moreover, the significant reduction of unknowns compared to domain discretization methods permits the use of a direct solver for most of our configurations.

Due to the difficulty in developing a stable BEM formulation for the wide range of frequencies and physical parameters, practical computation is usually based on a specific BEM formulation for each model. Indeed, the common Maxwell formulation (PMCHWT with RWG or Rooftop basis functions) suffers from numerical noise at low frequency or in presence of highly contrasted media. Hence, eddy current formulations [2], [3] are preferred for highly conductive bodies at low frequencies. They are generally considered as accurate for near field computations, but require the introduction of additional unknowns ($\mathbf{n}\cdot\mathbf{H}$ and possibly $\mathbf{n}\cdot\mathbf{E}$, in addition to $\mathbf{J} = \mathbf{n}\times\mathbf{H}$ and $\mathbf{M} = \mathbf{E}\times\mathbf{n}$). Besides, eddy current approximations are valid only for low frequencies.

II. Low-frequency or high-contrast reformulations

The PMCHWT system is given by

$$\mathbf{Z}\cdot\mathbf{X} = \mathbf{Y} \tag{1}$$

and, more explicitly, by

$$\begin{bmatrix} \mathbf{Z}_0^{JJ} + \mathbf{Z}_1^{JJ} & \mathbf{Z}_0^{JM} + \mathbf{Z}_1^{JM} \\ \mathbf{Z}_0^{MJ} + \mathbf{Z}_1^{MJ} & \mathbf{Z}_0^{MM} + \mathbf{Z}_1^{MM} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{X}^J \\ \mathbf{X}^M \end{bmatrix} = \begin{bmatrix} \mathbf{Y}^J \\ \mathbf{Y}^M \end{bmatrix}$$

where subscripts 1 and 0 refer to internal and external contributions (respectively related to a bounded body Ω and the surrounding air filling the complementary domain $\mathbb{R}^3 \setminus \overline{\Omega}$). The singularity of the double layer potentials $\mathbf{Z}_{0,1}^{JM}, \mathbf{Z}_{0,1}^{MJ}$ give rise to twisted identity operators which cancel by summing internal and external contributions. Each single-layer potential $\mathbf{Z}_{0,1}^{JJ}, \mathbf{Z}_{0,1}^{MM}$ is the sum of a vector potential and a scalar potential, which respectively behave like $O(\omega)$ and $O(\omega^{-1})$ in the low-frequency limit. The vector potential is thus in the numerical noise of the scalar potential at low frequencies. A popular way to overcome this low-frequency breakdown is to separate the two potentials through an Helmholtz decomposition (loop-tree basis functions) in (1) and to rescale the resulting system. The latter then has the form

$$\tilde{\mathbf{Z}}\cdot\tilde{\mathbf{X}} = \tilde{\mathbf{Y}} \tag{2}$$

with

$$\tilde{\mathbf{Z}} := \mathbf{N}_1\cdot(\mathbf{P}\cdot\mathbf{Z}\cdot\mathbf{P}^*)\cdot\mathbf{N}_2, \quad \tilde{\mathbf{X}} := \mathbf{N}_2^{-1}\cdot((\mathbf{P}^*)^{-1}\cdot\mathbf{X}), \quad \tilde{\mathbf{Y}} := \mathbf{N}_1\cdot(\mathbf{P}\cdot\mathbf{Y})$$

where $\mathbf{P}$ effects the change in basis functions (loop-tree). In partitioned form, system (2) has the form

$$\tilde{\mathbf{Z}}^{ab} = \begin{bmatrix} \mathbf{Z}_{LL}^{ab} & \mathbf{Z}_{LT}^{ab} \\ \mathbf{Z}_{TL}^{ab} & \mathbf{Z}_{TT}^{ab} \end{bmatrix}, \quad \tilde{\mathbf{X}}^b = \begin{bmatrix} \mathbf{X}_L^b \\ \mathbf{X}_T^b \end{bmatrix}, \quad \tilde{\mathbf{Y}}^a = \begin{bmatrix} \mathbf{Y}_L^a \\ \mathbf{Y}_T^a \end{bmatrix} \quad (a,b = J,M).$$

where subscripts L and T refer to loop and tree functions. The potentials $\mathbf{Z}_{LL}^{ab}, \mathbf{Z}_{LT}^{ab}, \mathbf{Z}_{TL}^{ab}$ involve only the original vector potentials, as the scalar potentials vanish when applied to, or tested with, loop basis functions. The diagonal weighting matrices $\mathbf{N}_{1,2}$, proposed in [4], perform a normalization based on the asymptotic expansion of the Green function for dielectric bodies (real wave number). Although a similar normalization could conceivably be defined for conductors, it would not be effective for the cases of present interest involving high contrasts between media 1 and 0. As an additional observation, the EC formulation introduced in [5] is retrieved from (1) by a procedure similar to that outlined above, namely neglecting the displacement current everywhere, applying the Helmholtz decomposition and suppressing the tree terms of the electric current density $\mathbf{J}_T$.

The above formulation does not address the difficulties caused by high contrasts. A popular approach for overcoming the large scaling due to high dielectric contrast consists in introducing suitable weighting factors to governing integral equations in both subdomains prior to combining them. This results in the new system

$$\hat{\mathbf{Z}} \cdot \mathbf{X} = \hat{\mathbf{Y}} \tag{3}$$

with

$$\hat{\mathbf{Z}}^{J\cdot}_{0,1} = \alpha_{0,1}\mathbf{Z}^{J\cdot}_{0,1}, \qquad \hat{\mathbf{Z}}^{M\cdot}_{0,1} = \beta_{0,1}\mathbf{Z}^{M\cdot}_{0,1},$$

$$\hat{\mathbf{Y}}^{J} = \alpha_0\mathbf{Y}^{J}, \qquad \hat{\mathbf{Y}}^{M} = \beta_0\mathbf{Y}^{M},$$

where suitable values of coefficients $\alpha_{0,1}, \beta_{0,1}$ are given in [6]. The twisted identity operators arising in the double layer potentials no longer cancel. The corresponding matrix is not diagonally dominant, which makes the set of RWG basis functions unsuitable as test functions. An alternative [6] consists in testing equations (3) with $\mathbf{n}\times$RWG basis functions. Recently, a variant has been proposed for highly conductive media and its association with Helmholtz decomposition seems to yield accurate results for far field computations at low frequencies [7]. Besides, a classical EC formulation can be retrieved by choosing $\alpha_0 = \beta_1 = 1$ and $\alpha_1 = \beta_0 = 0$, adding a scalar equation and neglecting the displacement current.

III. Numerical assessment

First, the bistatic radar cross section (RCS) of a dielectric sphere ($r = 0.5$m, $\epsilon_r = 5.0$, $f = 10$Hz) placed in the free space and illuminated by a plane wave [4] has been computed to validate decomposed (2) (LTN) and weighted (3) formulations for both RWG (W-RWG) and $\mathbf{n}\times$RWG (W-nxRWG) testing in the dielectric case. Both modified formulations lead to accurate results while the original PMCHWT formulation (1) suffers from the low-frequency breakdown, see Fig. 1.

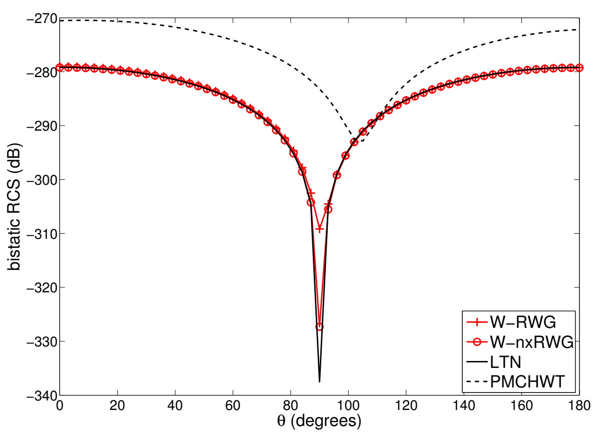

Figure 1: Bistatic RCS of a dielectric sphere.

Next, a configuration representative of ENDT experiments, hence involving a conductive body, is considered. Table I shows the variation of impedance of a coil ($r_{int} = 3$mm, $r_{ext} = 3.75$mm, $h = 2$mm, $I = 10^{-3}$A, 328 turns, $f = 100$Hz) placed 0.3mm above a conductive plate ($\sigma = 100$MS/m) large enough to neglect edge effects. Accurate results have been obtained for loop-tree decomposition, with normalization (LTN) or disregarding $\mathbf{J}_T$ (LLT), while PMCHWT suffers from low-frequency breakdown. However we also observed that similar configurations may exhibit instabilities, as the low-frequency and high-contrast aspects are not yet properly handled.

Table I: Variation of impedance $\Delta Z = Z_1 - Z_0$.

Reference	$0.01995 - 0.0068657i$
PMCHWT	$0.017514 + 0.064636i$
LTN	$0.019911 - 0.0068061i$
LLT	$0.019911 - 0.0068061i$

IV. Current investigation

To address these shortcomings, a perturbative approach is currently investigated, whereby the perturbation of a perfect electrical conductor (PEC) is considered. The PEC problem only involves the unknown $\mathbf{J}$ (or $\mathbf{J}_L$ under certain assumptions). Moreover, the resulting problem is expected no to suffer from scaling imbalance caused by high contrast, and to allow more accurate computation of internal fields in highly conductive bodies (like [8] for magnetostatics). This approach and preliminary results will be presented at the conference.

Our long-term objective is to establish a generic Maxwell formulation that can be modified in each sub-domain according to the characteristics of the corresponding medium. Such formulation is required to compute accurately the near field, even in highly conductive or permeable media, while keeping problem sizes reasonable.

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